Cascaded tanks benchmark combining soft and hard nonlinearities

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1 Introduction

Many systems exhibit a quasi linear or weakly nonlinear behavior during normal operation, and a hard saturation effect for high peaks of the input signal. The proposed benchmark is an example of this type of nonlinear system. On top of this, only a short data record is available for the parameter estimation step.

The next sections describe the cascaded tanks system (Section 2) and introduce the estimation and test data (Section 3). The figures of merit that are used in this benchmark are presented in Section 4. Finally, some of the expected challenges during the identification process are listed in Section 5.

2 Cascaded tanks system

The cascaded tanks system is a fluid level control system consisting of two tanks with free outlets fed by a pump. The input signal controls a water pump that pumps the water from a reservoir into the upper water tank. The water of the upper water tank flows through a small opening into the lower water tank, and finally through a small opening from the lower water tank back into the reservoir. This process is shown in Figure 1.

The relation between (1) the water flowing from the upper tank to the lower tank and (2) the water flowing from the lower tank into the reservoir are weakly nonlinear functions.

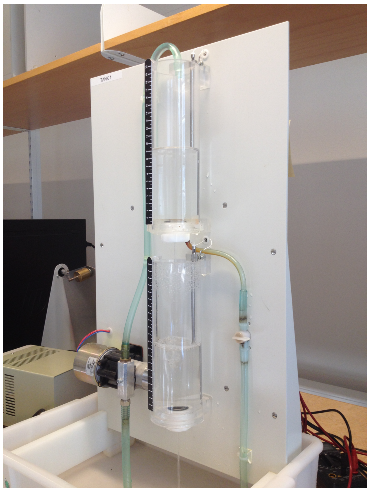

Figure 1: The cascaded tanks system: the water is pumped from a reservoir in the upper tank, flows to the lower tank and finally flows back into the reservoir. The input is the pump voltage, the output is the water level of the lower tank.

However, when the amplitude of the input signal is too large, an overflow can happen in the upper tank, and with a delay also in the lower tank. When the upper tank overflows, part of the water goes into the lower tank, the rest flows directly into the reservoir. This effect is partly stochastic, hence it acts as an input-dependent process noise source. The overflow saturation nonlinear behavior of the lower tank is clearly visible in the time domain representation of the output signals (see Figure 2). A video of such an overflow situation can be found on the benchmark website.

Without considering the overflow effect, the following input-output model can be constructed based on Bernoulli's principle and conservation of mass:

$$\dot{x}_1(t) = -k_1\sqrt{x_1(t)} + k_4 u(t) + w_1(t), \tag{1}$$

$$\dot{x}_2(t) = k_2\sqrt{x_1(t)} - k_3\sqrt{x_2(t)} + w_2(t), \tag{2}$$

$$y(t) = x_2(t) + e(t), \tag{3}$$

where $u(t)$ is the input signal, $x_1(t)$ and $x_2(t)$ are the states of the system, $w_1(t)$, $w_2(t)$ and $e(t)$ are additive noise sources, and k_1, k_2, k_3 and k_4 are constants depending on the system properties.

3 Estimation and test data

The input signals are multisine signals which are 1024 points long, and excite the frequency range from 0 to 0.0144 Hz, both for the estimation and test case. The lowest frequencies have a higher amplitude then the higher frequencies (see Figure 2). The sample period T_s is equal to 4 s. The input signals are zeroth-order hold input signals.

The process is controlled from a Matlab interface to the A/D and D/A converters attached to the water level sensor and the pump actuator. The water level is measured using

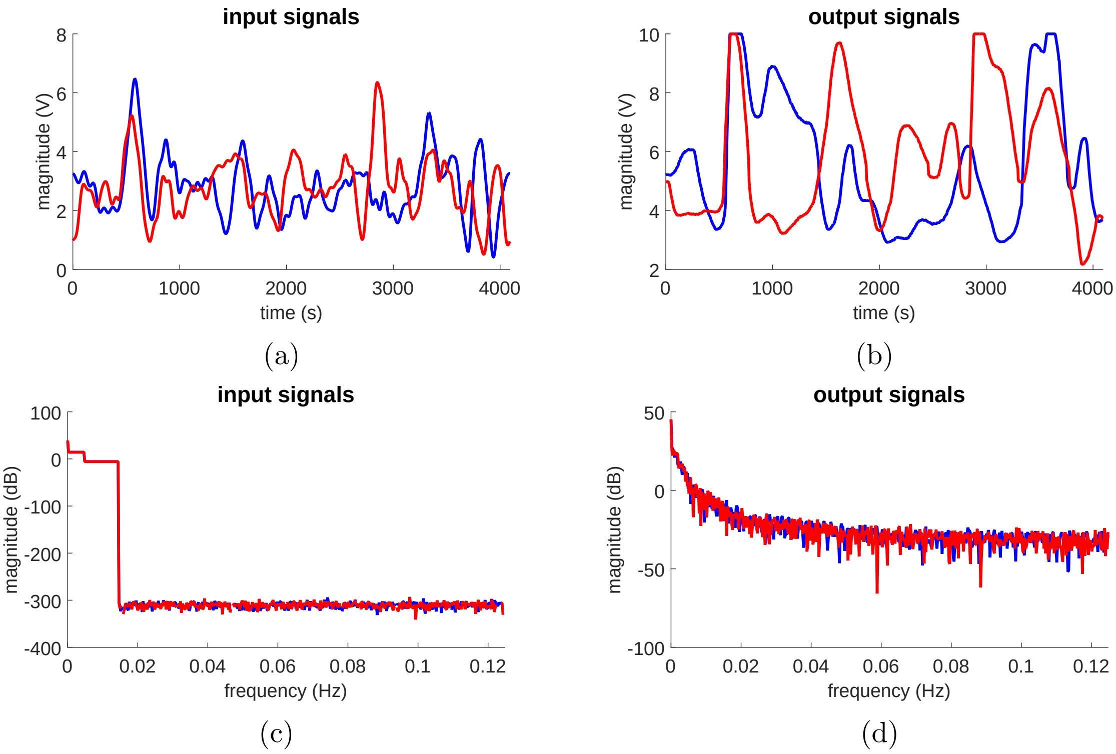

Figure 2: Input (a,c) and output (b,d) signals of the estimation (blue) and test (red) data records in the time (a,b) and frequency (c,d) domain.

capacitive water level sensors, the measured output signals have a signal-to-noise ratio that is close to 40 dB. The water level sensors are considered to be part of the system, they are not calibrated and can introduce an extra source of nonlinear behavior.

Note that the system was not in steady state during the measurements. The system states have an unknown initial value at the start of the measurements. This unknown state is the same for both the estimation and the test data record.

4 Figure of merit

The goal of the benchmark is to estimate a good model on the estimation data. The test data should not be used for any purpose during the estimation.

We expect all participants of the benchmark to report the following figure of merit for all test datasets to allow for a fair comparison between different methods:

$$\boldsymbol{e_{RMSt}} = \sqrt{1/N_v \sum_{t=1}^{N_t} (y_{mod}(t) - y_t(t))^2}, \tag{4}$$

where y_{mod} is the modeled output, y_t is the output provided in the test data set, N_t is the total number of points in y_t.

Also mention whether the modeled output y_{mod} is obtained using **simulation** (only the test input u_t is used to obtain the modeled output $y_{mod}(t) = F(u_t(1), \ldots, u_t(t))$) or **prediction** (both the test input u_t and the past test output y_t are used to obtain the modeled output $y_{mod}(t) = F(u_t(1), \ldots, u_t(t), y_t(1), \ldots, y_t(t-1))$). Provide both figures of merit (simulation and prediction) if the identified model allows for it.

5 Nonlinear system identification challenges

We anticipate the cascaded tanks benchmark to be associated with 4 major nonlinear system identification challenges:

- the hard saturation nonlinearity combined with the weakly nonlinear behavior of the system in normal operation,
- the overflow from the upper to the lower tank, this effect also introduces input-dependent process noise,
- the relatively short estimation data record,
- the unknown initial values of the states.