

This sheet is used in the proof of Lemma 11.

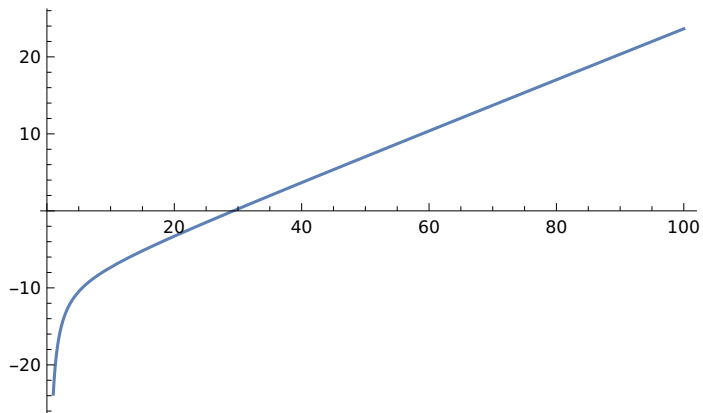
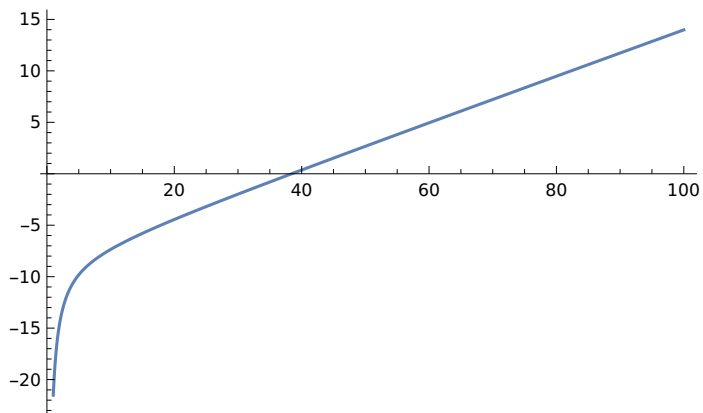
The goal is to show that $\text{excess}[9, m] > 0$ for $m > 30$ and $\text{excess}[10, m] > 0$ for $m > 40$.

```
In[4]:= rho = (1 + Sqrt[2]) / 2;  
beta = Sqrt[2] / (1 + Sqrt[2]);  
excess[k_, m_] := (4 / (3 * rho) - 1) * (k * m - k) - (1 - beta) * m -  
  1 / 4 - beta * m / 2 - 4 * beta * k / 3 - (3 * (m + 1) + 3) / (32 * k) - 3 * k / (2 * m)  
Table[N[Solve[excess[k, m] == 0 && m > 0, m]], {k, 9, 10}]
```

```
Out[7]= {{{m -> 38.4285}}, {{m -> 29.2738}}}
```

```
Plot[excess[9, m], {m, 1, 100}]
```

```
Plot[excess[10, m], {m, 1, 100}]
```



```
NSolve[excess[9, m] == 0, m]
```

```
NSolve[excess[10, m] == 0, m]
```

```
Out[12]= {{m -> 38.4285}, {m -> -1.5711}}
```

```
Out[13]= {{m -> 29.2738}, {m -> -1.55645}}
```