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Progressive Failure Analysis of composite Materials using the Puck Failure Criteria

Karan Kodagali
University of South Carolina

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PROGRESSIVE FAILURE ANALYSIS OF COMPOSITE MATERIALS USING THE
PUCK FAILURE CRITERIA

by

Karan Kodagali

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Accepted by:

Addis Kidane, Director of Thesis

Subramani Sockalingam, Reader

Cheryl L. Addy, Vice Provost and Dean of the Graduate School

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ABSTRACT

Fiber reinforced composites have been used in various engineering structures and applications especially in naval, automotive, aeronautical and sports industries. These composite materials generally exhibit brittle damage behavior. The anisotropy in the material and different kinds of failure mechanisms make it difficult to accurately characterize the behavior of composite materials. The present work aims to verify and apply the Puck Failure Criteria using the commercially available finite element package ABAQUS by writing a user-material subroutine in FORTRAN. The model is implemented with different post failure degradation schemes.

In the present work, the progressive failure on composite materials is analyzed using the Puck failure criteria to detect damage initiation. The ABAQUS user defined material subroutine UMAT was developed to apply the failure criteria and degradation models. The progressive failure analysis of a single lamina of a composite material is carried out on an open hole specimen under uniaxial tension. A partial discount method and a gradual stiffness degradation method is implemented and the results using these degradation models are compared. The damage initiation and progression obtained from the proposed model is compared with the observed experimental results and the digital image correlation data. This model was then used for the progressive failure analysis of a composite laminate with a central hole loaded in inplane tension with different stacking sequences and compared with the results obtained from literature.

From the results, it can be seen that the Puck failure hypothesis is a robust and versatile criteria which can be used for the progressive failure analysis of continuous fiber unidirectional composite laminates.

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LIST OF ABBREVIATIONS

DIC.....	Digital Image Correlation
FF	Fiber Failure
FEA	Finite Element Analysis
IFF	Inter Fiber Failure
UMAT	User Material Subroutine
WWFE	World Wide Failure Exercise

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Composite materials are materials made from two or more constituent materials with significantly different material properties combined to make a superior material with unique properties. Composites occur naturally, for example wood found in nature and even the bones in every skeletal system are composite materials. Composites have been used as building materials for thousands of years. Mud bricks have been reinforced with straw materials which provide more tensile strength than conventional mud bricks. Concrete is also a composite material, it is a mixture of aggregate, cement and sand. Most modern composites are made of two materials – fibers which provide strength and carry a bulk of the tensile load and a matrix or binder material to reinforce the fibers.

Recently fiber reinforced composites have been used in various engineering structures and applications especially in naval, automotive, aeronautical and sports industries. The Boeing 787 Dreamliner and the Airbus A380 are large capacity passenger airplanes and make use of composite materials owing to the high stiffness to low weight, high tensile strength, non-corrosive properties and the fact that composite materials have different properties in different directions, makes it possible for the materials to be tailor-made specifically for the product requirement. Carbon fiber reinforced composite materials can have up to five times the strength of 1020 grade steel while having one-fifth of the weight [2].

These composite materials generally exhibit brittle damage behavior. There is little plastic deformation and failure occurs suddenly. The anisotropy in the material and different kinds of failure mechanisms make it difficult to accurately characterize the behavior of composite materials.

A number of failure criteria were proposed to describe the damage in composite materials. Section 2 of this thesis deals with the literature review wherein a brief review of many of the progressive failure models are provided. In the early 90s, the World-Wide Failure Exercise (WWFE) was initialized and to provide a comprehensive coordinated study of the predictive capabilities of prominent failure criteria currently in use to describe the behavior of fiber reinforced laminates. The authors of many of the failure criteria were invited to provide blind predictions for different cases and then these predictions were evaluated against other predictions and the experimental data. The first exercise, WWFE I dealt with 2D stress cases with 19 failure criteria being evaluated [1]. The second exercise, WWFE II dealt with 3D stress cases with 12 criterions being evaluated [2]. The third exercise WWFE III dealt with laminates with a stress concentration under inplane loading conditions [3]. From the results of the WWFE I and WWFE II, there was no clear consensus on the best performing failure criteria for all the different load cases however, it showed the strengths and the shortcomings of the criteria. The Puck Failure Criteria was found to perform well for most of the test cases.

1.2 OBJECTIVE

Though the performance of composite materials is very good, it presents a challenge to develop composite structures for use in various industries. Numerical simulations can help to reduce cost and time for developing these structures. The present

work aims to apply the Puck Failure Criteria using the commercially available finite element package ABAQUS by writing a user-material subroutine in FORTRAN to simulate the progressive failure of continuous fiber unidirectional composite materials. The model is implemented with different post failure degradation schemes. The model is validated against an experiment conducted on a single layer lamina with a central hole loaded in inplane tension. The Digital Image Correlation (DIC) data and the experimental data were compared with the model prediction. Another validation test was conducted by comparing failure loads for a group of composite laminates with a central hole and different stacking sequences subjected to inplane tension.

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CHAPTER 2

LITERATURE REVIEW

This section provides a brief review of the various progressive failure models available in literature.

W Van Paepegem and J Degrieck [1] have proposed a residual stiffness model which simulates the full cycle from initial decline to final failure. The modified Tsai-Wu criterion by Tsai-Liu was further modified to determine the calculated safety factor and then defined the fatigue failure index which can be accepted as a suitable stress measure. The model was developed as two functions - damage initiation and damage propagation and the final layout of the model was a superposition of the two functions. The model developed is one-dimensional in nature, only longitudinal stiffness is considered and delamination's have not been included in the model.

C. Schuecker and H.E. Pettermann [2] proposed a continuum damage model based on brittle failure mechanisms. They hypothesized that any non-linear material behavior was the result of brittle cracks forming in the composites. Puck 2D criterion was employed to determine failure modes and damage growth. First, the current damage on the current load measure is computed and then, the effect of damage on the elasticity tensor is predicted by a fourth order tensor equation also taking into account the current stress state. The laminate response predicted by this model is too stiff under shear dominated loading conditions.

Mahmood M Shokrieh and Larry B Lessard [3] proposed a model capable of simulating the fatigue behavior of laminated composites under general loading conditions, with or without stress concentrations. This model can determine the residual strength, residual stiffness and fatigue life of composite laminates with arbitrary geometry and stacking sequence under complicated fatigue loading conditions. Failure modes were determined by Hashin's criteria.

S C Tan and R J Nuismer [4] proposed a progressive matrix cracking model in which the laminate is assumed to contain periodic cracks with even spacing. A plane stress assumption and a generalized plane stress assumption were employed. The salient feature about this model is that it requires basic material properties such as moduli, Poisson ratio, thermal expansion coefficients and the specific fracture energy. This model is only applicable under the given assumptions.

Fu-Kuo Chang and Kuo-Yen Chang [5] proposed a damage model for notched laminates subjected to tensile loads with any arbitrary ply orientations. The 2D plane stress assumption was used and loads were assumed to increase incrementally in small steps such that the stress-strain relations were assumed to be linear. A finite element method combined with a Newton-Raphson scheme was developed to solve the model.

F Cesari, V Dal Re, G Minak and A Zucchelli [6] have proposed a damage model for carbon-fiber reinforced epoxy-resin laminates loaded at the center to simulate low velocity impacts. The 3D Hashin criteria were used to determine the individual damage modes. A numerical model to predict the first ply failure and the ultimate ply failure of the laminate was developed using ANSYS.

Dahlen C and Springer G S [7] proposed a semi-empirical model to determine the growth of delamination in laminates under cyclic loading and mode I, mode II and mixed mode conditions. Mode III was assumed to not contribute significantly to delaminations. A growth law similar to Paris growth law was employed.

Xiao J and Bathias C [8] studied notched and un-notched woven composites with mechanical properties the warp direction being much higher than those in the weft. They showed that the ratios between fatigue strength and ultimate tensile strength for both notched and un-notched cases are equal to their respective static strength ratios.

H A Whitworth [9] proposed a model to predict the stiffness degradation in composite laminates based on an assumed relation between the failure stiffness and the applied stress. The statistical distribution of the residual stiffness is obtained from a 2-parameter Weibull distribution. The theoretical distribution over-predicts in some cases, the accuracy improves with increasing cycle number. The present model is only limited to specimens subjected to constant amplitude fatigue loading and assumes that the residual stiffness is a monotonically decreasing function of the fatigue cycles.

Alexandros E Antoniou, Christoph Kensche and Theodore P Philippidis [10] proposed 3 different models, one implements the Puck's failure criteria and associated progressive stiffness degradation rule, the second one is based on Lessard and Shokrieh limit theory while the third is similar to the first, implements Puck's IFF criteria and associated gradual stiffness degradation rules however having different conditions for fiber breakage. Failure modes were restricted to 2D in plane patterns. Elastic modulus in the fiber direction and the major Poisson ratio were considered constant.

P C Wang, S M Jeng and J M Yang [11] studied the stiffness reduction and evolution of microstructural damage of a unidirectional composite under tension-tension fatigue. A partial crack shear-lag model developed by Kuo and Chou for ceramics was adopted and modified for application in metal composites to predict residual stiffness as a function of fatigue damage evolution. The fiber matrix interfacial bonding was assumed to be perfect. The results suggest that the matrix crack density controls the stiffness degradation profile. The residual stiffness is independent of the applied stress levels however the accumulation of microstructural damage varies with the applied stress.

J N Yang, D L Jones, S H Yang and A Meskini [12] proposed a stiffness degradation model to predict the statistical distribution of the residual stiffness of composites subjected to fatigue loading. Two analytical methods were presented, one based on linear regression analysis and the other on the Bayesian approach. The results are only accurate if fatigue life data already exists up to 50% of the fatigue life.

K I Tserpes, P Papanikos and Th Kermanidis [13] proposed a 3-D progressive damage model to simulate the damage accumulation and predict the residual strength and final failure mode of bolted composite joints under in-plane tensile loading. The 3D Hashin failure criteria as reported by Shokrieh and Lessard was used to predict failure. Material property degradation rules as proposed by S C Tan were used.

T Kevin O'Brien and Kenneth L Reifsnider [14] proposed a secant modulus criterion that would predict fatigue failure of the laminate while tests were being carried out. When the static stiffness measured during fatigue, E_f degrades from its initial tangent modulus, E_i , to the secant modulus measured in a static ultimate strength test, E_s regardless of load history, fatigue failure occurs. The secant modulus criterion however was not a

valid failure criterion for general application but can be applied to only specific laminate orientations.

C D M Liljedahl, A D Crocombe, M A Wahab and I A Ashcroft [15] proposed a numerical modelling techniques for predicting the environmental degradation of adhesively-bonded joints. A CZM was implemented in the FEA by use of a user-defined element (UEL). The CZM parameters were determined by correlating experimental data and numerical predictions for initial failure loads.

W Hwang and K S Han [16] proposed a new concept called "fatigue modulus," which is defined as a slope of applied stress and resultant strain at a specific cycle. They assumed that the fatigue modulus degradation follows a power function of the fatigue cycles. The fatigue life was determined from the fatigue modulus and was found to have a better agreement with experimental data than the S-N curves and Basquin's relation.

Stephen R Hallet and Michael R Wisnom [17] proposed a new approach to modelling of notched composite materials using interface elements to model the inter and intra ply damage. This method was developed to model delamination in composites. It can be used to predict the initiation and propagation of the delamination, however it requires a prior knowledge of the potential failure sites.

Timothy W Coats and Charles E Harris [18] experimentally verified a continuum damage model which was used to predict the development of progressive damage in a toughened material system. The Allen and Harris model was employed to model the behavior of micro-crack damage by predicting stiffness loss and damage in a laminate. . The model neglects edge effects, uses internal state variables to represent the local deformation effects.

A S Koumpias, K I Tserpes and S Pantelakis [19] developed a progressive damage model to simulate the mechanical response, predict the quasi static strength of a fully interlaced 3D woven composite and predict the damage initiation and progression as a function of the applied load as well as the stiffness and strength of the composite. The Hashin type failure criteria were used to predict the failure modes. To model the mechanical response of the matrix, the multi-linear isotropic hardening material model developed by *Rolfes et al* was used in the progressive damage model. The predicted failure pattern for longitudinal tension is in complete agreement with the tests from that of Stig and Hallstrom

Yuang Liang, Hai Wang, Costas Soutis, Tristan Lowe and Robert Cernik [20] conducted quasi-static punch shear tests on satin weave carbon/epoxy laminates in an effort to determine the damage that could develop during a penetrating impact event. The Hashin criteria was employed as the failure criteria. Once damage occurs in an element based on the Hashin criteria, a ply-discount degradation of material property was applied as the damage progression strategy. A constant parameter, β_k was used as the damage variable in the stiffness reduction method. Using a function instead of a constant as the damage variable would yield better results.

M Ridha, C H Wang, B Y Chen and T E Tay [21] developed a progressive failure model for orthotropic composite laminates to predict the effect of specimen size and laminate orthotropy on the open-hole tension (OHT) strength. The max stress failure criterion is combined with the Tsai-Wu failure criterion to model fiber-dominated and matrix-dominated failure. The models are able to predict the correct trend of the effect of specimen size on OHT strength, however the model under predicts the OHT strength of a

specimen having four 0° plies because the stiffness and strength of laminates increases as the percentage of 0° plies increases.

John Montesano, Marina Selezneva, Martin Levesque and Zouheir Fawaz [22] developed a fatigue prediction model to predict damage tolerance capability of polymer matrix composite structures. The model accounts for local multi-axial as well as variable amplitude cyclic loading. The continuum damage model (CDM) developed also incorporates a cumulative damage law that is a function of the number of loading cycles. The model assumes that during unloading, the material properties are same as the undamaged materials, suitable failure criteria's are not defined. The model also assumes that compressive stresses do not cause any damage and thus do not affect material stiffness.

Ciaran R Kennedy, Conchur M O Bradaigh and Sean B Leen [23] presented a model that combines the fatigue induced fiber strength and modulus degradation, irrecoverable cyclic strain effects and inter fiber fatigue. The predicted response captures the overall modulus degradation in the first cycle and the evolution of degradation in subsequent cycles until failure however, delamination was not considered as a failure mode.

C T McCarthy , R M O'Higgins and R M Frizzell [24] developed a novel approach where a cubic spline interpolation method was used to capture the non-linear shear behavior. A ply discount method based of Hashin's criteria was employed to determine the damage, also the spline approach along with the maximum strain failure criteria was employed to predict the shear response. This model accurately predicts tensile strength and modulus but under-predicts the ultimate transverse strain. But only when shear stresses dominate

Brett A Bednaryck, Bertram Stier, Jaan-W Simon and Evan J Pineda [25] presented a comparison between the meso scale and micro scale approaches to modelling progressive damage in plain weave reinforced polymer matrix composites. A continuum damage model was developed and implemented based on the 2-D approach given by Barbero and was extended for 3-D case. The damage model is based on the principle of energy equivalence with infinitesimal strains. The micromechanics model was based on the generalized method of cells (GMC) developed by Paley and Aboudi. It is an efficient semi-analytical method that provides homogenized, non-linear constitutive response of a composite material. Very similar results were obtained using the two approaches, however these models were not compared with any experimental results.

Bartley-Cho J, Lim S G, Hahn h T and Shyprykevich P [26] studied the behavior of quasi-isotropic graphite epoxy laminates. The authors obtained a failure function which varies with number of cycles following an experimentally determined relationship to predict ply cracking. The crack density was calculated, and it was found that in absence of other competing damage modes, the crack density increased with applied load levels which is opposite to the belief that crack density is independent of load history.

Talreja R [27] presented a continuum damage model where internal damage variable are characterized by tensorial quantities. Matrix cracking and delamination were the only damage modes considered, and it was assumed that these damage modes do not mutually interact but were accounted for separately one damage mode at a time and the effects were later superimposed.

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CHAPTER 3

THEORY

3.1 SCALE OF THE ANALYSIS

The analysis of the composite materials can be conducted at four different scale – micro-mechanical, lamina level, laminate level and structural level [1]. The micromechanical level considers the fibers and matrix separately, each having different properties and different behavior. The interaction between the fibers and matrix is considered. In the lamina level, the fibers and matrix are treated as homogeneous anisotropic materials. Orthotropic material models are generally used at this scale. Most of the failure criteria are developed at this level and are considered in a layer-wise manner for intralaminar failure [2]. On the laminate level, the material is observed as a stack of several laminae, including interfaces. At this level, inter-laminar and intra laminar stresses are obtained for each layer and also for the interfaces. Intra laminar failure analysis and delamination is conducted at this level. On the structural level, whole components of the structure are considered. These may involve complex local stacking sequences and geometries of the component. In the current work, the Puck failure criteria is utilized at the lamina scale and delamination failure has not been considered.

The constitutive models relate the state of strain to the state of stress. The model used in a three-dimensional material model for a linear elastic and orthotropic material. The normal components are coupled while the shear components are completely uncoupled.

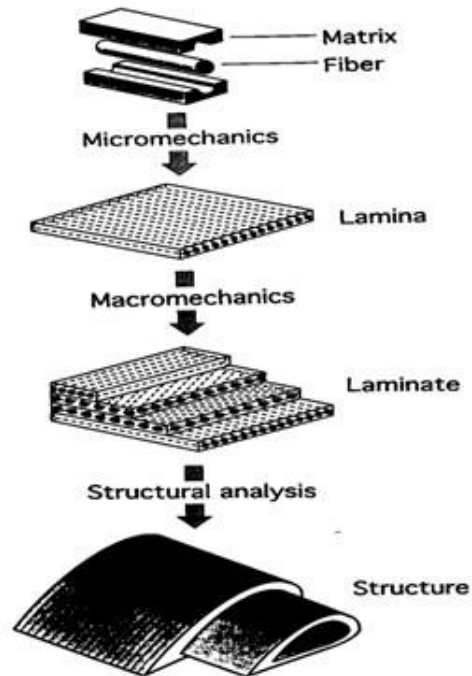


Figure 3.1 Different Scales of Analysis

3.2 PUCK FAILURE CRITERIA

Fiber reinforced composites usually display brittle fracture mechanics wherein the fracture occurs suddenly without major plastic deformation. The macroscopic failure of a composite can be seen at the lamina scale. This appears as fiber fracture (FF) or inter fiber fracture (IFF). The Puck theory presents separate equations for the FF and IFF.

3.2.1 FIBER FAILURE

The fiber failure generally is regarded as the final failure of the lamina. Fiber failure is defined as the simultaneous breakage of a large number of elementary fibers [3]. The fibers have a much higher stiffness than the matrix and carries much higher loads in the fiber direction. However, transverse to the fiber direction, nearly the same amount of stress acts on both the fiber and the matrix. The fiber failure is considered as a statistical process. Individual fibers may start to fail at 60% of the fiber fracture limit for static load cases [4].

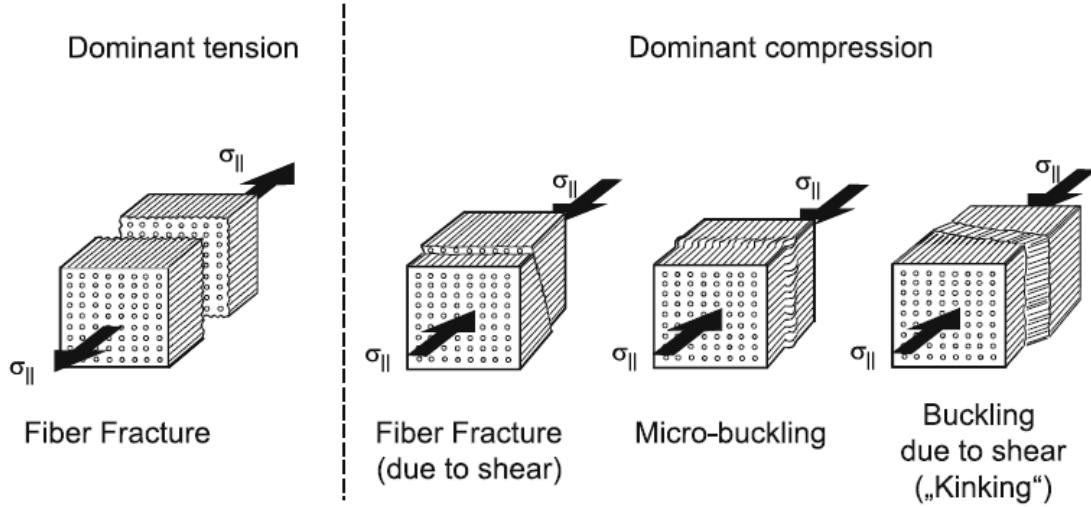


Figure 3.2 Different forms of Fiber Fracture (FF)

Figure 3.2 illustrates the different fiber fracture modes. Under a tensile load, the fibers rupture perpendicular to the fiber direction. Under a compressive load, three failure modes are possible. Buckling is the prominent damage mode wherein the fibers in a large region bend in a common direction, with fiber kinking being the buckling on a more macroscopic level. Fiber fracture due to shear rarely occurs. It requires a perfect alignment of the fibers and bonding of the fiber-matrix in which case shear stresses acting on the fibers causes the fracture at an inclined fracture plane. The fiber failure impedes the ability of the lamina to carry load and causes delamination's and stress concentrations in nearby laminas which may lead to subsequent failures [5].

The fiber failure is generally caused by $\sigma_{||}$ stresses. In the earlier versions of the Puck failure criteria [6], a maximum stress criterion was used to describe the fiber failure as shown in the equations below.

$$f_{E,FF} = \frac{\sigma_1}{\pm R_{||}^{t,c}} \quad (3.1)$$

Where σ_1 is the tensile stress along the fiber direction and $R_{\parallel}^{t,c}$ are the tensile and compressive strengths of the material. R_{\parallel}^t is used for positive σ_1 and $-R_{\parallel}^c$ is used for negative σ_1 .

However, for a more accurate analysis the effects of σ_2 and σ_3 have to be considered. Due to different Youngs moduli for the fiber and matrix, though the stress is similar the micro-mechanical strain is different. A stress magnification factor, $m_{\sigma f}$ for the transverse stresses takes this into account. Puck proposed a value of 1.3 for GFRP and 1.1 for CFRP. This discrepancy is due to the fact that glass fibers have a higher Youngs modulus than carbon fibers [7].

According to Puck, the fiber failure occurs when the stress in the fibers σ_{1f} reaches the strength of the fibers [8]. Thus, the Puck criteria uses the stresses and strengths of the fibers instead of the material. This is similar to the maximum stress criteria but is extended to the fibers. The equation for the fiber failure is derived as follows:

$$\varepsilon_{1f} = \frac{\sigma_{1f}}{E_{\parallel f}} - \frac{\nu_{\parallel\perp f}}{E_{\perp f}} m_{\sigma f} (\sigma_2 + \sigma_3) \quad (3.2)$$

Using $\frac{\nu_{\parallel\perp f}}{E_{\perp f}} = \frac{\nu_{\parallel f}}{E_{\parallel f}}$, and $\varepsilon_{1f} = \varepsilon_1$, the equation can be rearranged for σ_{1f}

$$\sigma_{1f} = E_{\parallel f} \varepsilon_1 + \nu_{\parallel f} m_{\sigma f} (\sigma_2 + \sigma_3) \quad (3.3)$$

Also, $\varepsilon_1 = \frac{\sigma_1}{E_{\parallel}} - \frac{\nu_{\parallel\perp}}{E_{\parallel}} (\sigma_2 + \sigma_3)$ and $\sigma_{1f \text{ at fracture}} = \pm R_{\parallel f}^{t,c} = \frac{E_{\parallel f}}{E_{\parallel}} \pm R_{\parallel}^{t,c}$

$$f_{E,FF} = \frac{1}{\pm R_{\parallel}^{t,c}} \left[\sigma_{11} - \left(\nu_{\perp\parallel} - \nu_{\perp\parallel f} \cdot m_{\sigma f} \frac{E_{\parallel}}{E_{\parallel f}} \right) (\sigma_{22} + \sigma_{33}) \right] \quad (3.4)$$

$$\text{with } \begin{cases} +R_{\parallel}^t & \text{for } [\dots] \geq 0 \\ -R_{\parallel}^c & \text{for } [\dots] \leq 0 \end{cases}$$

Where $f_{E,FF}$ is the fiber failure stress exposure of the lamina, $\pm R_{\parallel f}^{t,c}$ are the effective tensile and compressive strengths of the fiber parallel to fiber direction, $\pm R_{\parallel}^{t,c}$ is the tensile and compressive strengths of the material, σ_{11} , σ_{22} and σ_{33} are the normal stresses acting in the lamina, $\nu_{\perp\parallel}$ and $\nu_{\perp\parallel f}$ are the major Poisson's ratio of the lamina and of the fibers respectively, E_{\parallel} and $E_{\parallel f}$ are the longitudinal modulus of the lamina and the fibers respectively and $m_{\sigma f}$ is the stress magnification factor for transverse stresses in the fibers.

For purely tensile loading, this criterion performs similar to the maximum stress criteria but under higher transverse stresses, the effect is more pronounced. Fiber failure based on the Puck theory is the last ply failure of the laminate.

3.2.2 INTER FIBER FRACTURE

Inter fiber failure or matrix failure can be defined as a macroscopic crack formation through the matrix material. It includes the cohesive matrix fracture and the adhesive fracture of the fiber-matrix-interphase. An IFF crack is generated instantly and propagates till the fiber boundaries. The IFF can occur in different forms as seen in Figure 3.3 based on the kind of loading. Under transverse tension or longitudinal shear, a straight crack oriented perpendicular to the stress is observed while under transverse compression and transverse shear, an inclined crack is observed. The presence of IFF leads to a redistribution of stresses in the laminate but the lamina is still able to carry some load. The presence of IFF leads to successive damage due to a concentration of the stresses and to delamination, especially near the crack tip.

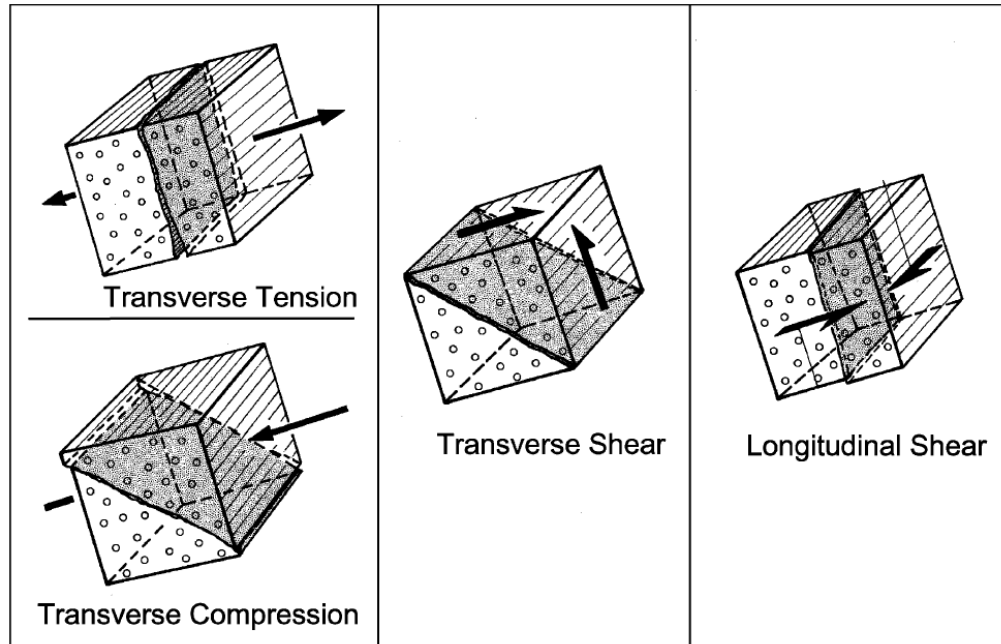


Figure 3.3 Different forms of Inter Fiber Fracture (IFF)

An IFF can have a varying impact on the capacity of a laminate depending on the angle of the fracture plane. The straight cracks formed under transverse tension or longitudinal shear can generally be tolerated. The major risk with this type of damage is the growth of delamination at the crack tips and the damage accumulation due to stress concentrations around the crack. On the other hand, the inclined fracture angle under transverse compression and transverse shear loadings are usually destructive for the laminate. It leads to high instantaneous delamination's and even the splitting of the laminate and may lead to the wedge effect.

The Puck failure theory determines the angle of the fracture plane and uses the stresses acting on this plane to determine IFF. It is based on the formulations of Coulumb and Mohr. The Mohr hypothesis states that the fracture limit of a material is determined by the stresses acting on the fracture plane. This was originally stated for brittle isotropic materials and was adapted by Puck for the transversely orthotropic brittle composite

materials. The fracture plane is oriented parallel to the fiber direction and at an angle Θ to the thickness direction. The stresses acting on this action plane σ_n , τ_{n1} and τ_{nt} are used to determine the IFF. The shear stresses τ_{n1} and τ_{nt} can be combined to form the shear stress $\tau_{n\psi}$.

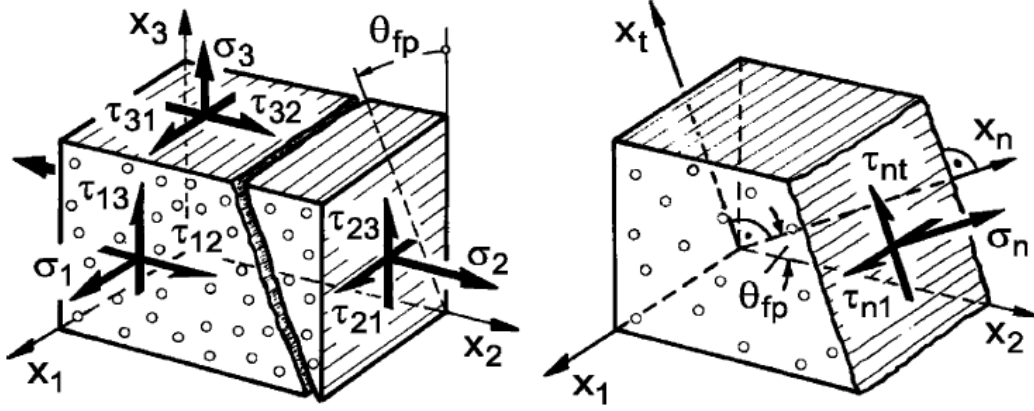


Figure 3.4 Stresses acting on the fracture plane

These stresses are obtained from transforming the σ_2 , σ_3 , τ_{21} , τ_{31} and τ_{23} stresses as seen from the following:

$$\begin{Bmatrix} \sigma_n(\theta) \\ \tau_{nt}(\theta) \\ \tau_{n1}(\theta) \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc & 0 & 0 \\ -sc & sc & (c^2 - s^2) & 0 & 0 \\ 0 & 0 & 0 & s & c \end{bmatrix} \begin{Bmatrix} \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{21} \end{Bmatrix} \quad (3.5)$$

Where,

$$c = \cos\theta \quad \text{and} \quad s = \sin\theta$$

The Puck IFF criterion can be written as follows:

For $\sigma_n \geq 0$:

$$f_{E_{IFF}}(\theta) = \sqrt{\left[\left(\frac{1}{R_{\perp}^{At}} - \frac{p_{\perp\psi}^t}{R_{\perp\psi}^A}\right)\sigma_n(\theta)\right]^2 + \left(\frac{\tau_{nt}(\theta)}{R_{\perp\perp}^A}\right)^2 + \left(\frac{\tau_{n1}(\theta)}{R_{\perp\parallel}^A}\right)^2} + \frac{p_{\perp\psi}^t}{R_{\perp\psi}^A}\sigma_n(\theta) \quad (3.6)$$

For $\sigma_n < 0$:

$$f_{E_{IFF}}(\theta) = \sqrt{\left(\frac{\tau_{nt}(\theta)}{R_{\perp\perp}^A}\right)^2 + \left(\frac{\tau_{n1}(\theta)}{R_{\perp\parallel}^A}\right)^2 + \left(\frac{p_{\perp\psi}^c}{R_{\perp\psi}^A} \sigma_n(\theta)\right)^2} + \frac{p_{\perp\psi}^c}{R_{\perp\psi}^A} \sigma_n(\theta) \quad (1.7)$$

Where,

$$\begin{aligned} \frac{p_{\perp\psi}^{t,c}}{R_{\perp\psi}^A} &= \frac{p_{\perp\perp}^{t,c}}{R_{\perp\perp}^A} \cos^2 \psi + \frac{p_{\perp\parallel}^{t,c}}{R_{\perp\parallel}^A} \sin^2 \psi \\ \cos^2 \psi &= 1 - \sin^2 \psi = \frac{\tau_{nt}^2}{\tau_{nt}^2 + \tau_{n1}^2} \\ R_{\perp\perp}^A &= \frac{R_{\perp}^c}{2(1 + p_{\perp\perp}^c)} \end{aligned}$$

R_{\perp}^{At} and $R_{\perp\parallel}^A$ are the tensile strength perpendicular to fiber direction and the in-plane shear strength respectively, $R_{\perp\perp}^A$ is the fracture resistance due to transverse/transverse shear stressing. θ_{fp} is the angle of the fracture plane and $p_{\perp\parallel}^{t,c}, p_{\perp\perp}^{t,c}$ are inclination parameters. $f_{E_{IFF}}$ is the failure effort or stress exposure of the inter fiber failure of the lamina. When the value of $f_E = 1$ is reached, it is termed as the fracture condition of the lamina. If σ_n is a tensile stress it promotes IFF by assisting the shear stresses but if σ_n is a compressive stress it delays IFF by raising the fracture resistances against shear fracture. Therefore, separate equations are used to evaluate IFF under tensile and compressive σ_n [8].

The action plane orientated at the angle θ_{fp} is the fracture plane, this is the angle at which the highest risk of fracture occurs. This angle is determined by calculating $f_{E_{IFF}}(\theta)$ for all planes with angles ranging from $\theta = -90^\circ$ to $\theta = 90^\circ$ with 1° steps, and the plane with the largest stress exposure is the plane where fracture is to be expected.

$$[f_{E_{IFF}}(\theta)]_{max} = f_{E_{IFF}}(\theta_{fp}) \quad (1)$$

The inclination parameters $p_{\perp\parallel}^{t,c}, p_{\perp\perp}^{t,c}$ are obtained from the (σ_{22}, τ_{21}) curves. However, it is difficult to obtain these parameters without doing a series of experiments to obtain this and thus Puck provided recommended values for these inclination parameters as listed below:

Table 3.1: Recommended Values For Inclination Parameters

	$p_{\perp\parallel}^c$	$p_{\perp\parallel}^t$	$p_{\perp\perp}^{t,c}$
GFRP	0.25	0.30	0.20-0.25
CFRP	0.30	0.35	0.25-0.30

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CHAPTER 4

PROGRESSIVE FAILURE ANALYSIS OF A COMPOSITE LAMINA

USING PUCK FAILURE CRITERIA ¹

¹ Kodagali K, Tessema A, Kidane A. American Society of Composites, 2017

4.1 ABSTRACT

This paper focuses on the progressive failure analysis of a composite at lamina scale using two different material property degradation models in an open hole specimen under uniaxial tension. Puck failure criterion is selected to detect the onset of damage. The ABAQUS user defined material subroutine UMAT was developed to apply the failure criteria and degradation models. A partial discount method and a gradual stiffness degradation is implemented in ABAQUS environment. The damage initiation and progression obtained from the proposed model is compared with the observed experimental results from digital image correlation. The comparative study confirmed that the simulation results were in good agreement with the experimental results.

Keywords: Progressive failure analysis, Stiffness Degradation, Puck failure criteria, DIC

4.2 INTRODUCTION

Fiber-reinforced composites have been established as competitive materials for naval, automotive and aerospace industry during the last few decades. Their high strength to low weight ratio attracts a lot of attention to applying it in different industries. Therefore, it is important to understand the deformation behavior and failure mechanisms of these of materials under mechanical loading. An imperative part of material behavior is the concept of damage. Most of the current failure criteria are developed at the lamina scale and hence it is necessary to see the effect of damage in this scale. A good understanding of initiation and propagation of damage in composites will help to predict the strength of the structure at a higher accuracy. There are a lot of experimental and FEM analysis about the damage

propagation at the laminate scale; however little work has been carried out explicitly at the lamina scale.

There are a lot of failure criteria that have been developed over the past few decades. The most general failure criterion for composite materials is the Tensor Polynomial Criterion proposed by Tsai and Wu [1]. The other popular and well known failure criteria include those proposed by Tsai-Hill [2], Azzi-Tsai [3], Hoffman [4] and Chamis [5]. These criteria do not consider the heterogeneous nature of a lamina and do not provide the type of failure. The other type of failure criteria which considers the non-homogeneous characteristics of the composites can be used to differentiate the failure modes in the material. Hart Smith proposed a generalized Tresca model which considers fiber shearing as a dominant failure mode [6]. Hashin-Rotem proposed a criterion that involves two failure mechanisms, one associated with fiber failure and the other with matrix failure, distinguishing between tension and compression [7].

There are other failure criteria including those by, S C Tan and R J Nuismer who proposed a progressive matrix cracking model in which the laminate is assumed to contain periodic cracks with even spacing [8]. H A Whitworth proposed a model to predict the stiffness degradation in composite laminates based on an assumed relation between the failure stiffness and the applied stress [9]. Yamada and Sun proposed a criterion considering the in situ shear strength coupled with the probabilistic nature of composite failure [10]. R. M. Christensen proposed a criterion where micromechanics was used to distinguish failure modes [11].

Puck and Schürmann built on the Hashin failure criteria, the fiber failure (FF) was dependent on material properties of the fiber instead of the properties of the ply, and the

inter-fiber failure (IFF) was differentiated into three including the transverse tension (mode A), moderate transverse compression (mode B), and large transverse compression (mode C). Also, an equation was proposed to determine the angle of the fracture plane [12]. Both the Hashin and the Puck and Schürmann criteria were 2D criteria ignoring interlaminar stresses.

Later, Puck modified his criteria to include interlaminar stresses. The new failure criteria was termed as action-plane failure criteria wherein IFF was calculated based on stresses which act on planes parallel to the fiber and inclined at an angle θ with respect to the thickness direction [13].

The Verein Deutscher Ingenieure (VDI) provides a detailed description of the concepts and design of composites and the analysis using the Puck failure criteria and are incorporated in this work [14].

Once the damage is found to initiate by the failure criteria, the material properties are degraded to simulate the presence of cracks in the material. One of the popular and simple method is the Total Discount Method [15] wherein the stiffness are reduced to zero in the failed ply. Further work on stiffness reduction has been developed by Nahas [16] and Soden [17] among others. In the current work, a partial discount method [18] and a gradual stiffness reduction method [12-13] are utilized to model the degradation.

In order to validate the Puck damage prediction model, the evolution of damage in a specimen must be observed. One approach is to utilize a non-destructive technique that will allow for the detection of damage evolution in a composite structure such as digital image correlation (DIC). In our work experiments are conducted with the help of DIC which provides full field deformation data on the specimen surface.

4.3 THEORY

4.3.1 CONSTITUTIVE RELATION

The constitutive models in a quasi-static stress analysis relates the state of strain to the state of stress. The material is considered as linear elastic and transversely orthotropic. The linear elastic stress–strain constitutive relation can be written as follows:

$$\{\epsilon\} = [S]\{\sigma\} \text{ or } \{\sigma\} = [C]\{\epsilon\} \quad (4.1)$$

$$C_{11} = E_{11}(1 - \nu_{23}\nu_{32})\Delta \quad (4.2)$$

$$C_{22} = E_{22}(1 - \nu_{13}\nu_{31})\Delta \quad (4.3)$$

$$C_{33} = E_{33}(1 - \nu_{12}\nu_{21})\Delta \quad (4.4)$$

$$C_{12} = E_{11}(\nu_{21} - \nu_{31}\nu_{23})\Delta \quad (4.5)$$

$$C_{13} = E_{22}(\nu_{32} - \nu_{12}\nu_{31})\Delta \quad (4.6)$$

$$C_{23} = E_{33}(\nu_{31} - \nu_{21}\nu_{32})\Delta \quad (4.7)$$

$$\Delta = 1 / (1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{21}\nu_{32}\nu_{13}) \quad (4.8)$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2G_{23} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \epsilon_{12} \\ \epsilon_{13} \\ \epsilon_{23} \end{Bmatrix} \quad (4.9)$$

Where C_{ij} are the material stiffness tensors ν_{ij} are the Poisson ratio, E_{ij} are the Young's moduli and G_{ij} are the shear moduli

4.3.2 PUCK FAILURE THEORY

Puck's criteria for fiber fracture (FF) and inter-fiber fracture (IFF) of unidirectional reinforced composites are physically based on hypotheses and mathematical formulations appropriate for brittle fracture. The formulations of Coulomb [19], Mohr [20] and Paul [21]

are particularly important, which have been developed for quasi-isotropic materials. They have been adapted by Puck to the transversely orthotropic UD fiber/polymer composites.

The action-plane fracture criteria is formulated using stresses σ_n , τ_{nt} and τ_{n1} instead of σ_{11} , σ_{22} , σ_{33} , τ_{12} , τ_{13} , τ_{23} , which act on a plane parallel to the fibers and at an angle θ . These stresses are calculated with the aid of the following transformation:

$$\begin{Bmatrix} \sigma_n(\theta) \\ \tau_{nt}(\theta) \\ \tau_{n1}(\theta) \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc & 0 & 0 \\ -sc & sc & (c^2 - s^2) & 0 & 0 \\ 0 & 0 & 0 & s & c \end{bmatrix} \begin{Bmatrix} \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{21} \end{Bmatrix} \quad (4.10)$$

Where,

$$c = \cos\theta \quad \text{and} \quad s = \sin\theta$$

In order to characterize certain types of stress, Puck introduces the concept of ‘stressing’ [22], differentiating stresses into acting transverse (\perp) to the fiber direction or parallel (\parallel) to the fiber direction.

The Puck failure criterion can be written as follows:

$$f_{\text{EFF}} = \frac{1}{\pm R_{\parallel}^{\text{tc}}} \left[\sigma_{11} - \left(v_{\perp\parallel} - v_{\perp\parallel\text{f}} \cdot m_{\text{of}} \frac{E_{\parallel}}{E_{\parallel\text{f}}} \right) (\sigma_{22} + \sigma_{33}) \right] \quad (4.11)$$

$$\text{with} \begin{cases} +R_{\parallel}^{\text{t}} & \text{for } [\dots] \geq 0 \\ -R_{\parallel}^{\text{c}} & \text{for } [\dots] \leq 0 \end{cases}$$

For $\sigma_n \geq 0$:

$$f_{\text{EFF}}(\theta) = \sqrt{\left[\left(\frac{1}{R_{\perp}^{\text{At}}} - \frac{p_{\perp\psi}^{\text{t}}}{R_{\perp\psi}^{\text{A}}} \right) \sigma_n(\theta) \right]^2 + \left(\frac{\tau_{nt}(\theta)}{R_{\perp\perp}^{\text{A}}} \right)^2 + \left(\frac{\tau_{n1}(\theta)}{R_{\perp\parallel}^{\text{A}}} \right)^2} + \frac{p_{\perp\psi}^{\text{t}}}{R_{\perp\psi}^{\text{A}}} \sigma_n(\theta) \quad (4.12)$$

For $\sigma_n < 0$:

$$f_{E_{IFF}}(\theta) = \sqrt{\left(\frac{\tau_{nt}(\theta)}{R_{\perp\perp}^A}\right)^2 + \left(\frac{\tau_{n1}(\theta)}{R_{\perp\parallel}^A}\right)^2 + \left(\frac{p_{\perp\psi}^c}{R_{\perp\psi}^A} \sigma_n(\theta)\right)^2} + \frac{p_{\perp\psi}^c}{R_{\perp\psi}^A} \sigma_n(\theta) \quad (4.13)$$

Where,

$$\begin{aligned} \frac{p_{\perp\psi}^{t,c}}{R_{\perp\psi}^A} &= \frac{p_{\perp\perp}^{t,c}}{R_{\perp\perp}^A} \cos^2 \psi + \frac{p_{\perp\parallel}^{t,c}}{R_{\perp\parallel}^A} \sin^2 \psi \\ \cos^2 \psi &= 1 - \sin^2 \psi = \frac{\tau_{nt}^2}{\tau_{nt}^2 + \tau_{n1}^2} \\ R_{\perp\perp}^A &= \frac{R_{\perp}^c}{2(1 + p_{\perp\perp}^c)} \end{aligned}$$

Where, σ_{11} , σ_{22} and σ_{33} are normal stresses in the lamina, $\pm R_{\parallel}^{t,c}$ are the tensile and compressive strengths parallel to fiber direction, $v_{\perp\parallel f}$ is the fiber volume fraction, $m_{\sigma f}$ is the stress magnification factor, $E_{\parallel f}$ is the Young's modulus of the fiber along the fiber direction, R_{\perp}^{At} is the tensile strength perpendicular to fiber direction, $R_{\perp\parallel}^A$ is the in-plane shear strength, $R_{\perp\perp}^A$ is the resistance offered against fracture due to transverse/ transverse shear stressing, θ_{fp} is the angle of the fracture plane and $p_{\perp\parallel}^{t,c}, p_{\perp\perp}^{t,c}$ are inclination parameters obtained from the (τ_{n1}, σ_n) and (τ_{nt}, σ_n) fracture curves respectively. f_E is called the stress exposure and it is the ratio between the length of the vector of the stresses $\{\sigma\}$ and the length of the corresponding fracture vector $\{\sigma\}_{fr}$ which have the same direction. When $f_E = 1$, it is termed as the fracture condition.

If σ_n is a tensile stress it assists the shear stresses in causing IFF, and if σ_n is a compressive stress it delays IFF due to the shear stresses. Therefore, separate equations are used to evaluate IFF [23].

The stress exposure $f_{E_{IFF}}(\theta_{fp})$ is dependent on the angle of the fracture plane θ . This angle is determined by calculating $f_{E_{IFF}}(\theta)$ for angles ranging from $\theta = -90^\circ$ to $\theta = 90^\circ$, and the plane with the largest stress exposure is the plane where fracture is to be expected.

$$[f_{E_{IFF}}(\theta)]_{\max} = f_{E_{IFF}}(\theta_{fp}) \quad (4.14)$$

The angle of the fracture plane is important in assessing failure, for example $\theta_{fp} \approx 90^\circ$ indicates a high probability of delamination, if $\theta_{fp} > 30^\circ$ and $\sigma_n < 0$ it indicates Mode C failure. The fracture plane in the mode C is different and it would be destructive for the laminate. One surface of the crack slides over the other surface of crack causing the local delamination or buckling of neighbors called the wedge effect [14].

The values of the inclination parameters recommended by Puck are used in the current work as shown in Table 4.1 [24].

Table 4.1: Recommended values for inclination parameters

	$p_{\perp\parallel}^c$	$p_{\perp\parallel}^t$	$p_{\perp\perp}^{t,c}$
GFRP	0.25	0.30	0.20-0.25
CFRP	0.30	0.35	0.25-0.30

4.3.3 MATERIAL PROPERTIES DEGRADATION

As the damage identified in the composite material, the appropriate approach should be applied to assess the damage growth until failure. Two methods have been compared for the material degradation after the damage was initiated. The first method is called element weakening method which is a partial discount method [19]. The second

method is the constant stress exposure method which is a gradual stiffness reduction method [12-13]. These degradation methods are defined in a smeared crack approach, wherein the degradation process is assumed as the growth of the crack density.

4.3.3.1 ELEMENT WEAKENING METHOD (EWM)

In this method, selected elastic properties of elements are degraded to zero when the stress exposure reaches a value of one. The modulus perpendicular to fibers and inplane shear modulus are degraded for IFF and all of the elastic properties are degraded in case of a FF. However, as this causes convergence issues in the FE software the values are instead degraded to a value close to zero. Depending on the failure mode the values of the damage variables are updated. For fiber failure under tension $d_{ft}=1$, under compression $d_{fc}=1$; for matrix failure, due to $\sigma_n > 0$, $d_{mt}=1$, due to $\sigma_n < 0$, $d_{mc}=1$. The degradation rule for EWM used is as listed below:

$$d_f = (1 - d_{ft})(1 - d_{fc}) \quad (4.15)$$

$$d_m = (1 - d_{mt})(1 - d_{mc}) \quad (4.2)$$

$$C'_{11} = (1 - d_f)C_{11} \quad (4.3)$$

$$C'_{22} = (1 - d_f)(1 - d_m)C_{22} \quad (4.18)$$

$$C'_{33} = (1 - d_f)(1 - d_m)C_{33} \quad (4.19)$$

$$C'_{12} = (1 - d_f)(1 - d_m)C_{12} \quad (4.20)$$

$$C'_{13} = (1 - d_f)(1 - d_m)C_{13} \quad (4.21)$$

$$C'_{23} = (1 - d_f)(1 - d_m)C_{23} \quad (4.22)$$

$$G'_{12} = (1 - d_f)(1 - d_{mc}s_{mc})(1 - s_{mt}d_{mt})G_{12} \quad (4.23)$$

$$G'_{23} = (1 - d_f)(1 - d_{mc}s_{mc})(1 - s_{mt}d_{mt})G_{23} \quad (4.24)$$

$$G'_{31} = (1 - d_f)(1 - d_{mc}s_{mc})(1 - s_{mt}d_{mt})G_{31} \quad (4.25)$$

Where d_f and d_m are the total damage variables for the fiber and matrix respectively; d_{ft} , d_{fc} , d_{mt} , and d_{mc} are the fiber and matrix damage variables in relation to the tensile and compressive stress states, respectively; and s_{mt} and s_{mc} are the loss control factors for the shear stiffness caused by the matrix tensile and compressive failures, respectively. In the present study, the loss control factors were set as $s_{mt} = 0.9$ and $s_{mc} = 0.5$.

4.3.3.2 CONSTANT STRESS EXPOSURE METHOD (CSE)

The gradual stiffness reduction method is based on the stress exposure f_E . In this method when fiber failure occurs all the stiffness' are degraded like the EWM approach and only the stiffness due to IFF are gradually reduced. This approach conserves the meaning of the value $f_{E\ IFF} = 1$ as a fracture criterion, i.e. the lamina does not experience an IFF stress exposure above 1. When the stress exposure of a lamina reaches this value an IFF occurs and the lamina gets rid of parts of its load by redistribution. This is accomplished by incrementally degrading the stiffness based on the mode of damage to a value such that $f_{E\ IFF}$ will be equal to 1. The stiffness E_i and G_{ij} are degraded unequally [23]. The progressive damage rule for CSE is:

$$E_2^{\text{red}} = \begin{cases} E_2^{\text{orig}} \cdot (1 - d_{mt}) & \text{for } \sigma_n > 0 \\ E_2^{\text{orig}} & \text{for } \sigma_n < 0 \end{cases} \quad (4)$$

$$G_{12}^{\text{red}} = G_{12}^{\text{orig}} \cdot (1 - k \cdot d_{mt})(1 - k \cdot d_{mc}) \quad (4.27)$$

$$G_{23}^{\text{red}} = G_{23}^{\text{orig}} \cdot (1 - k \cdot d_{mt})(1 - k \cdot d_{mc}) \quad (4.28)$$

4.4 ANALYSIS

4.4.1 EXPERIMENTAL SETUP

A single zero-degree lamina with a central hole was used with dimensions given in Table 4.2 and configurations as shown in Figure 1. The specimen was loaded in uniaxial tension using an MTS810 machine with a loading rate of 0.05"/min. The material properties are listed in Table 4.3. DIC was used to capture the far-field global strain data. A 5MP camera with a 60mm Nikon lens was used to capture the gradual deformation of the speckled lamina during loading. A white light illumination is used to illuminate the speckled surface. The load at failure was determined to be 2805N.

Table 4.2: Dimensions of the Test Specimen

Length	Width	Thickness	Diameter of notch
200	25	0.22	4

Table 4.3: Material Properties Of Test Specimen

$E_{11}(GPa)$	82.867
$E_{22}(GPa)$	6.98
$G_{12}(GPa)$	13.24
V_{12}	0.306
V_{23}	0.28
V_{f12}	0.2
$E_{f1}(GPa)$	130
$R_{\parallel}^t(MPa)$	860
$R_{\parallel}^c(MPa)$	620
$R_{\perp}^t(MPa)$	37
$R_{\perp}^c(MPa)$	75
$R_{\perp\parallel}(MPa)$	44.7

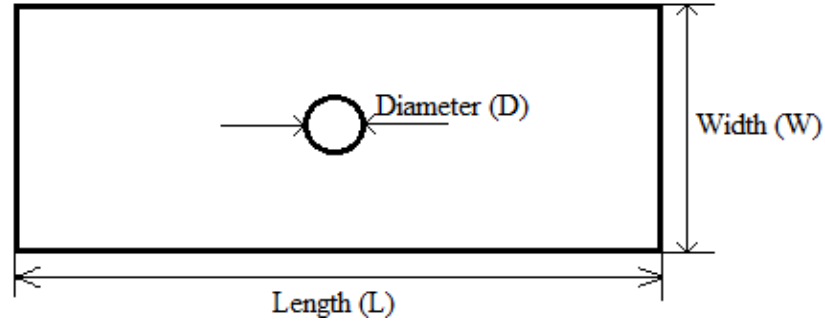


Figure 4.1 Test specimen Configuration.

4.4.2 COMPUTATIONAL ANALYSIS PROCESS

The commercial FEA software, ABAQUS was employed to simulate the model by using a user defined material behavior (UMAT subroutine). The composite was modeled with a load of 3000N being applied at one end and encastre boundary condition at the other end. Three-dimensional hexahedral element (C3D8R) was selected for FE simulation with a total of 15082 elements in the model. A small time step is used to help reduce nonlinearities in each step and hence improve the convergence.

Figures 2 shows the algorithm for the two degradation models used. This procedure is carried out at all material calculation points of elements for each increment. The initial strain and the incremental strain are received from ABAQUS along with the material properties. If damage is noted at the point the elastic properties are appropriately degraded. After which the stiffness matrix and the resulting stresses are calculated and then used as input parameters for the Puck theory to detect damage. If the fracture condition is met, material properties are degraded and this process continues until $f_E \leq 1$ or the maximum degradation is reached. After this, the Jacobian and state variables are updated and these values are returned to ABAQUS.

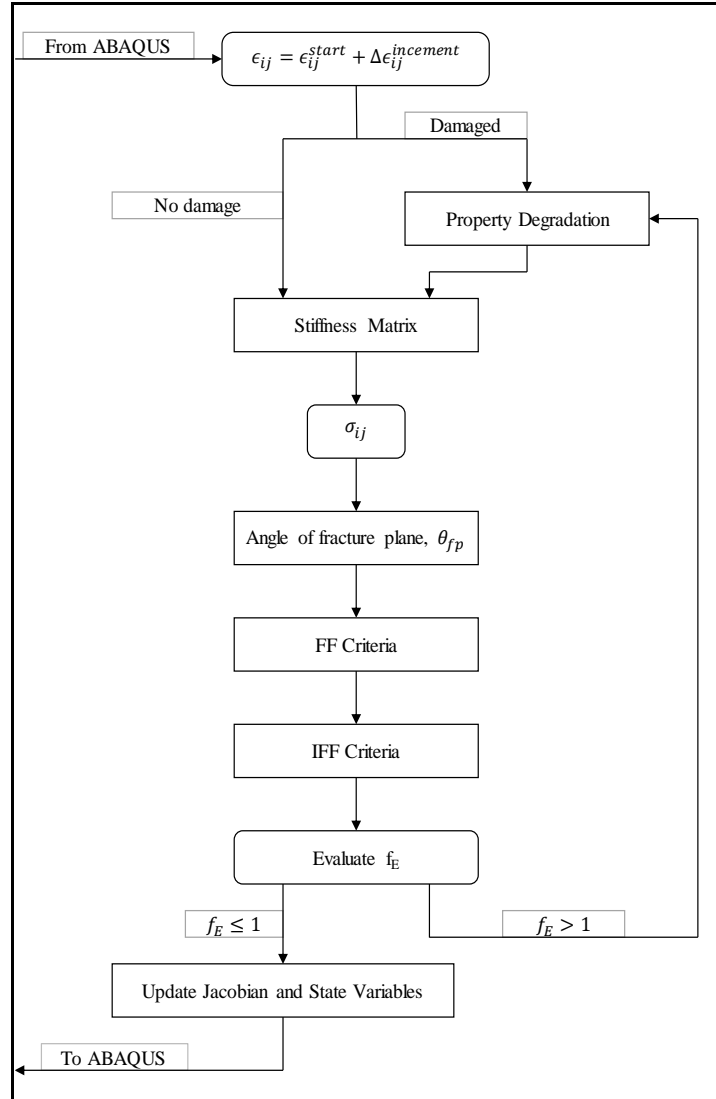


Figure 4.2 Algorithm for ABAQUS subroutine

4.4.3 RESULTS AND DISCUSSION

The strain distribution from DIC and FEM are shown in Figure 3. The strains from the FEM are compared with the DIC a few steps before failure at a load of 2508N. A line of elements around the hole are concealed from the FE results to match the DIC condition. The finite element strain distribution is in good agreement with the experimental results

for the CSE method. The EWM method shows much higher strains as fiber failure is detected at a previous load state.

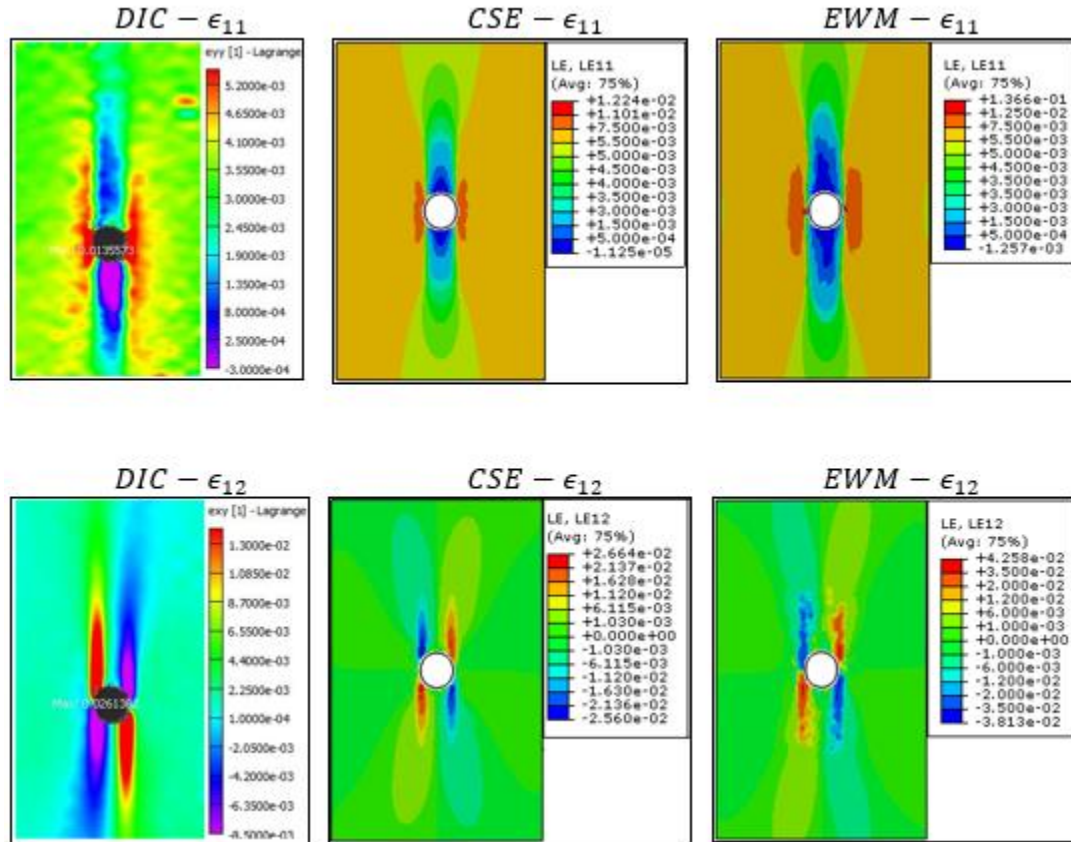


Figure 4.3 Experimental and Finite Element strain distribution at 2508N.

The results of the progressive damage from Puck criteria using the element weakening and constant stress exposure methods at increasing loading states are shown in Figure 4 and Figure 5 respectively. Initial matrix damage was observed at a load of 507.2N due to σ_n under tension. The matrix damage progressively increases tangential to the hole along the fiber direction in both the models. The contour of the damage due to EWM spreads more than the damage in the CSE. The partial discount degradation of EWM causes more elements to fail at a faster rate than the gradual degradation of CSE as the stiffness are instantly reduced causing higher strains to develop in the material. Final failure is

assumed at the first fiber breakage. For EWM fiber failure occurs at 2455N (12.4% error) while for CSE fiber failure occurs at 2637N (5.9%). Both the methods underpredicted the failure load but are within a reasonable limit.

From these results, it can be perceived that matrix cracks occur in the sample and once fiber failure occurs the crack propagates along the damaged matrix. The damage from the experimental results can be seen in Figure 6. Both the models show good agreement with the experimental results. The CSE method provides more accurate strain correlation with the DIC.

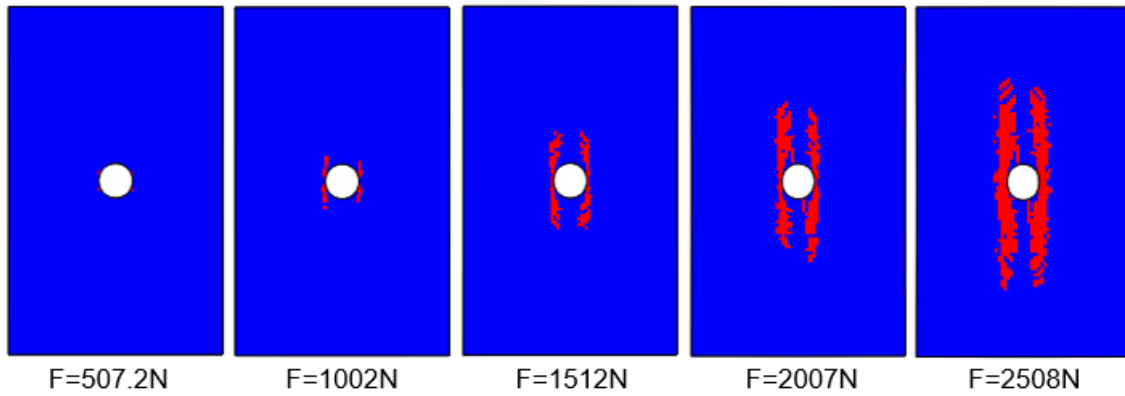


Figure 4.4 Matrix damage propagation at different load steps using EWM.

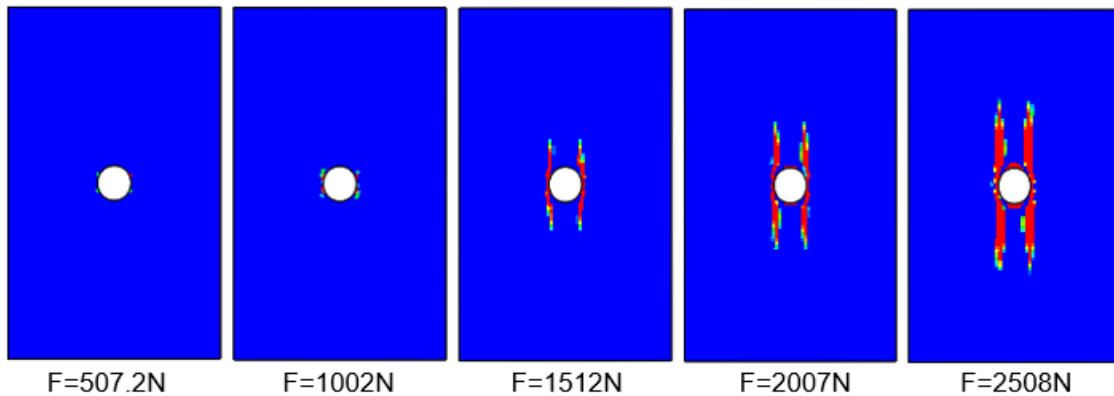


Figure 4.5 Matrix damage propagation at different load steps using CSE.

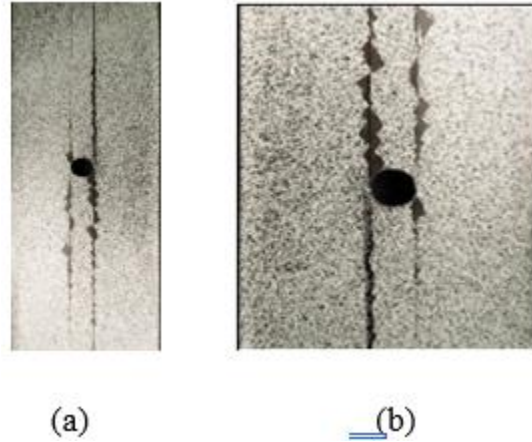


Figure 4.6 Image taken after failure of 0-degree specimen (b) close-up of specimen.

4.5 CONCLUSION

In the current study, the Puck failure criteria is used to predict the initiation and propagation of damage in 0 degree CFRP lamina using two degradation methods. The failure criterion and degradation models are implemented in ABAQUS using a UMAT subroutine. The FEA results are in good agreement with the experimental results. The gradual degradation method is found to be better at predicting failure of the composite specimen with an error of 5.9%. However, CSE is computationally more expensive as it requires to find the amount by which to degrade the element when an IFF occurs whereas EWM directly degrades the material to the maximum degradation. The Puck criteria is a robust criterion which can be employed as a to estimate damage initiation and progression in composite laminates and structures.

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CHAPTER 5

PROGRESSIVE FAILURE ANALYSIS OF A COMPOSITE LAMINATE USING THE PUCK FAILURE CRITERIA²

² Kodagali K, Kidane A. In Preparation

5.1 ABSTRACT

During the last few decades fiber reinforced composites have been used in various engineering structures and applications, especially their use in the naval, automotive and aerospace industries. These composite materials have high tensile strength and are lightweight, non-corrosive and can be tailored specifically for the product requirement. A safe design of such materials needs an adept and robust failure prediction under various loading conditions.

There have been a lot of failure criteria's proposed in literature. A recent study has found that the industrial usage of composite failure criteria's is limited to some of the simpler criteria's inbuilt into the finite element software's but these are unable to accurately capture damage and failure in the material [1]. The failure criteria can be broadly classified into criteria which do not distinguish between the different types of failure modes and those which do associate with a failure mode. All polynomial and tensorial failure criteria do not associate with any failure mode. The most commonly used polynomial failure criteria is the one proposed by Tsai and Wu [2]. Other prominent quadratic failure criteria include those by Tsai-Hill [3], Azzi-Tsai [4], Hoffman [5] and Chamis [6]. These criteria do not consider the heterogeneous nature of composite materials. These global fracture criteria are usually based on the von-Mises yielding criteria and can only be regarded as an interpolating formula as FRP usually show brittle fracture without major plastic deformation. The maximum stress and maximum strain criteria are the simplest criteria which takes into account the different failure modes. Other prominent criteria which differentiate failure modes include those by Hart-Smith [7], Yamada-Sun [8], Hashin-Rotem [9], Christensen [10], Puck [11] and Cuntze [12].

The Hashin failure criteria separated failure in the fiber and failure in the matrix under tensile and compressive stresses. Puck and Schürmann modified the Hashin criteria, the fiber failure (FF) depended on the properties of the fiber instead of the properties of the ply, the inter fiber failure (IFF) was separated into three equations including transverse tension, moderate transverse compression and large transverse compression. An equation to identify the angle of the fracture plane was also proposed. Puck based this failure criteria on the mechanics of brittle fracture. Both the Hashin and the Puck and Schürmann criteria were 2D criteria ignoring the effects of interlaminar stresses in the laminae

Later, Puck included interlaminar stresses and the new 3D Puck failure criteria was coined the action-plane failure criteria. In this the IFF was calculated using stresses action on a fracture plane which act on a plane parallel to the fibers and at angle θ to the thickness direction. The Puck criteria helps to identify the different failure modes in the lamina and to quantify the effect each type of failure has on the laminate [13].

When coupled with adequate degradation models, the Puck criteria can provide a good prediction of progressive failure and load redistribution. Once failure is initiated, the material properties are degraded to simulate the presence of cracks in the material. The presence of FF or IFF does not usually indicate the final failure of the specimen, but as damage accumulates in the material, it eventually fails. There have been a number of degradation models developed for damage in composites. The degradation procedure can be applied using damage mechanics or phenomenological approaches. Damage mechanics approaches generally have a representation of the damage and the law for the growth of the damage. The model by Li, Reid and Soden is an example of the damage mechanics based degradation. Phenomenological models are based on a stress/strain analysis instead of a

damage representation law. Total and partial ply discount are an example of phenomenological models wherein after IFF the parameters are degraded to zero based on the mode of damage.

The Verein Deutscher Ingenieure (VDI-2014 part3) (English: Association of German Engineers) provides a comprehensive description of the design of composites and failure analysis and degradation using the Puck failure criteria. Many of the recommendations from this are implemented in this work.

The World Wide Failure Exercise (WWFE-I and WWFE-II) conducted by Hinton, Kaddour and Soden invited the authors of many of the prominent failure criteria to take part in and compare the capabilities and the limitations of their criteria. A number of test cases and guidelines were provided and the participants provided the failure predictions based on their criteria. The organizers then provided the experimental data to validate the predictions. The organizers then analyzed and compiled all the results [14]. The Puck criteria was found to perform very well in almost all cases.

In the current study, the damage initiation and progression was carried out using the Puck failure criteria. The property degradation was carried out using the gradual stiffness degradation method. This model was implemented in the commercial finite element program ABAQUS using a user defined subroutine UMAT to implement the Puck criteria and property degradation. A model for unidirectional composite materials involves the examination of the constitutive relation to relate the state of stress to the state of strain, a failure criteria to determine initiation of damage in conjunction with a damage progression law for the evolution of damage in the material.

In order to validate the model, progressive failure analysis of composite laminates with a central hole is carried out under in-plane tensile loading. The results of these analysis are compared with the models from Tan [15], Chang and Chang [16] and J.F. Chen[17]. The results from the Puck failure criteria are found to be in good agreement with the test data reported in literature.

5.2 THEORY

5.2.1 CONSTITUTIVE RELATION

A linear elastic, transversely orthotropic stress-strain constitutive response is used in the model. The constitutive model is used to calculate the stress in the material using the material properties and the current strain in the material. The constitutive model is as follows:

$$\{\epsilon\} = [S]\{\sigma\} \text{ or } \{\sigma\} = [C]\{\epsilon\} \quad (5.1)$$

$$C_{11} = E_{11}(1 - \nu_{23}\nu_{32})\Delta \quad (5.2)$$

$$C_{22} = E_{22}(1 - \nu_{13}\nu_{31})\Delta \quad (5.3)$$

$$C_{33} = E_{33}(1 - \nu_{12}\nu_{21})\Delta \quad (5.4)$$

$$C_{12} = E_{11}(\nu_{21} - \nu_{31}\nu_{23})\Delta \quad (5.5)$$

$$C_{13} = E_{22}(\nu_{32} - \nu_{12}\nu_{31})\Delta \quad (5.6)$$

$$C_{23} = E_{33}(\nu_{31} - \nu_{21}\nu_{32})\Delta \quad (5.7)$$

$$\Delta = 1 / (1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{21}\nu_{32}\nu_{13}) \quad (5.8)$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2G_{23} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \epsilon_{12} \\ \epsilon_{13} \\ \epsilon_{23} \end{Bmatrix} \quad (5.9)$$

Where C_{ij} are the material stiffness tensors ν_{ij} are the Poisson ratio, E_{ij} are the Young's moduli and G_{ij} are the shear moduli.

5.2.2 PUCK FAILURE THEORY

The Puck failure criteria has been developed for transversely orthotropic unidirectional laminates. The concept of 'stressing' is introduced to characterize the types of stress. The stress are differentiated into those acting transverse (\perp) to the fiber direction or parallel (\parallel) to the fiber direction. The Fiber Failure (FF) of a lamina under a combined load occurs when the stress in the fibers reaches the value of the stress at a FF of the lamina under uniaxial tensile or compressive stresses. Fiber failure is the breakage of a large number of fibers and is generally regarded as final failure of the damaged lamina. The fiber failure criteria can be written as follows:

$$f_{E_{FF}} = \frac{1}{\pm R_{\parallel f}^{t,c}} \left[\sigma_{11} - \left(\nu_{\perp \parallel} - \nu_{\perp \parallel f} \cdot m_{\sigma f} \frac{E_{\parallel}}{E_{\parallel f}} \right) (\sigma_{22} + \sigma_{33}) \right] \quad (5.10)$$

$$with \begin{cases} +R_{\parallel}^t \text{ for } [...] \geq 0 \\ -R_{\parallel}^c \text{ for } [...] \leq 0 \end{cases}$$

Where $f_{E_{FF}}$ is the fiber failure stress exposure of the lamina, $\pm R_{\parallel f}^{t,c}$ are the effective tensile and compressive strengths of the fiber parallel to fiber direction, σ_{11} , σ_{22} and σ_{33} are the normal stresses acting in the lamina, $\nu_{\perp \parallel}$ and $\nu_{\perp \parallel f}$ are the major Poisson's ratio of the lamina and of the fibers respectively, E_{\parallel} and $E_{\parallel f}$ are the longitudinal modulus of the lamina and the fibers respectively. $m_{\sigma f}$ is a stress magnification factor for transverse stresses in the fibers. The values proposed by Puck for the magnification factor are $m_{\sigma f} = 1.1$ for CFRP and $m_{\sigma f} = 1.3$ for GFRP, this is because the Young's modulus for glass fibers is higher than that of carbon fibers. The relation between the effective strengths and the material strengths is given as:

$$\pm R_{\parallel f}^{t,c} = \frac{E_{\parallel f}}{E_{\parallel}} \pm R_{\parallel}^{t,c} \quad (5.11)$$

Puck's Inter Fiber Failure (IFF) is based on the formulation of Paul [18], Coulomb [19] and Mohr [20]. FRP usually display brittle behavior, fracture occurs suddenly without major plastic deformation. Paul called such material 'intrinsically brittle'. Mohr's fracture hypothesis for brittle materials states that the fracture limit of a material is determined by the stresses acting on the fracture plane. This was originally stated for brittle isotropic material, Puck adapted this to model transversely orthotropic unidirectional laminas. This action-plane fracture criteria is formulated using stresses σ_n , τ_{nt} and τ_{n1} which act on a plane parallel to the fibers and at an angle θ to it. These stresses are calculated with the assistance of the following transformation:

$$\begin{Bmatrix} \sigma_n(\theta) \\ \tau_{nt}(\theta) \\ \tau_{n1}(\theta) \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc & 0 & 0 \\ -sc & sc & (c^2 - s^2) & 0 & 0 \\ 0 & 0 & 0 & s & c \end{bmatrix} \begin{Bmatrix} \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{31} \\ \tau_{21} \end{Bmatrix} \quad (5.12)$$

Where, $c = \cos\theta$ and $s = \sin\theta$

The Puck IFF criterion can be written as follows:

For $\sigma_n \geq 0$:

$$f_{E_{IFF}}(\theta) = \sqrt{\left[\left(\frac{1}{R_{\perp}^{At}} - \frac{p_{\perp\psi}^t}{R_{\perp\psi}^A} \right) \sigma_n(\theta) \right]^2 + \left(\frac{\tau_{nt}(\theta)}{R_{\perp\perp}^A} \right)^2 + \left(\frac{\tau_{n1}(\theta)}{R_{\perp\parallel}^A} \right)^2} + \frac{p_{\perp\psi}^t}{R_{\perp\psi}^A} \sigma_n(\theta) \quad (5.13)$$

For $\sigma_n < 0$:

$$f_{E_{IFF}}(\theta) = \sqrt{\left(\frac{\tau_{nt}(\theta)}{R_{\perp\perp}^A}\right)^2 + \left(\frac{\tau_{n1}(\theta)}{R_{\perp\parallel}^A}\right)^2 + \left(\frac{p_{\perp\psi}^c}{R_{\perp\psi}^A} \sigma_n(\theta)\right)^2} + \frac{p_{\perp\psi}^c}{R_{\perp\psi}^A} \sigma_n(\theta) \quad (5.14)$$

Where,

$$\begin{aligned} \frac{p_{\perp\psi}^{t,c}}{R_{\perp\psi}^A} &= \frac{p_{\perp\perp}^{t,c}}{R_{\perp\perp}^A} \cos^2 \psi + \frac{p_{\perp\parallel}^{t,c}}{R_{\perp\parallel}^A} \sin^2 \psi \\ \cos^2 \psi &= 1 - \sin^2 \psi = \frac{\tau_{nt}^2}{\tau_{nt}^2 + \tau_{n1}^2} \\ R_{\perp\perp}^A &= \frac{R_{\perp}^c}{2(1 + p_{\perp\perp}^c)} \end{aligned}$$

R_{\perp}^{At} and $R_{\perp\parallel}^A$ are the tensile strength perpendicular to fiber direction and the in-plane shear strength respectively, $R_{\perp\perp}^A$ is the fracture resistance due to transverse/transverse shear stressing. θ_{fp} is the angle of the fracture plane and $p_{\perp\parallel}^{t,c}, p_{\perp\perp}^{t,c}$ are inclination parameters. $f_{E_{IFF}}$ is the failure effort or stress exposure of the inter fiber failure of the lamina. When the value of $f_E = 1$ is reached, it is termed as the fracture condition of the lamina.

If σ_n is a tensile stress it promotes IFF by assisting the shear stresses but if σ_n is a compressive stress it delays IFF by raising the fracture resistances against shear fracture. Therefore, separate equations are used to evaluate IFF under tensile and compressive σ_n [21].

The action plane orientated at the angle θ_{fp} is the fracture plane, this is the angle at which the highest risk of fracture occurs. This angle is determined by calculating $f_{E_{IFF}}(\theta)$ for all planes with angles ranging from $\theta = -90^\circ$ to $\theta = 90^\circ$ with 1° steps, and the plane with the largest stress exposure is the plane where fracture is to be expected.

$$[f_{E_{IFF}}(\theta)]_{\max} = f_{E_{IFF}}(\theta_{fp}) \quad (5.15)$$

The angle of the fracture plane is important in assessing failure, for if $\theta_{fp} > 30^\circ$ and $\sigma_n < 0$ it indicates Mode C failure. The fracture plane in the mode C is different and it would be destructive for the laminate. One surface of the crack slides over the other surface of crack causing the local delamination or buckling of neighbors called the wedge effect [22].

The values of the inclination parameters recommended by Puck are used in the current work as shown in Table I [23].

Table 5.1: Recommended values for inclination parameters

	$p_{\perp\parallel}^c$	$p_{\perp\parallel}^t$	$p_{\perp\perp}^{t,c}$
GFRP	0.25	0.30	0.20-0.25
CFRP	0.30	0.35	0.25-0.30

According to Mohr's hypothesis, σ_1 does not influence IFF as the action plane inclined at θ_{fp} is perpendicular to σ_1 . However, some effects make it necessary to include σ_1 in the IFF criteria as sometimes fibers will have fractured before the FF limit is reached, microfractures in the fiber may also occur leading to local debonding at the fiber matrix interphase. To include these effects a degradation factor due to the weakening caused by σ_1 , η_{w1} is multiplied to the action plane fracture resistances leading to higher stress exposure values. Therefore, the stress exposure is divided by η_{w1} to obtain the failure effort including the influence of σ_1 .

$$\eta_{w1} = \frac{c(a\sqrt{c^2(a^2 - s^2)} + 1 + s)}{(ca)^2 + 1}$$

With $c = \frac{f_{E0}(IFF)}{f_E(FF)}$ and $a = \frac{1-s}{\sqrt{1-m^2}}$, for the current work $s=m=0.5$

5.2.3 MATERIAL PROPERTIES DEGRADATION

The Puck failure criteria is stress based failure criteria that describes the maximum bearable stress state in a layer. When the stress exposure value reaches one, failure is assumed

to occur in the material. Damage in the material is simulated using the smeared crack representation whereby the cracks are not discrete local discontinuities but only their effect on the material property is considered. The presence of cracks in a mechanical sense is the reduction of the stiffness of the lamina. Thus, a degradation of the elastic moduli due to the presence of cracks is carried out which leads to a redistribution of stresses. The Constant Stress Exposure (CSE) method is the degradation law used in this model. The degradation process is assumed as the growth of crack density in the material.

5.2.3.1 CONSTANT STRESS EXPOSURE METHOD (CSE)

The constant stress exposure method is a gradual stiffness degradation method based on the failure effort, f_E . This method conserves the failure condition of $f_{E\ IFF} = 1$, i.e. the lamina at no point experiences a stress exposure value greater than one. When the stress exposure reaches or exceeds this value, load is redistributed in the lamina by incrementally increasing the degradation until the value of the failure effort is equal to one. The stiffness E_i and G_{ij} are unequally degraded by a factor ‘k’ [24]. The suggested values for ‘k’ are, k=77% for CFRP and k=41% for GFRP. The progressive damage rule for CSE is:

$$E_i^{\text{red}} = \begin{cases} E_i^{\text{orig}} \cdot (1 - d_{mt}) & \text{for } \sigma_n > 0 \\ E_i^{\text{orig}} & \text{for } \sigma_n < 0 \end{cases} \quad (5.5)$$

$$G_{ij}^{\text{red}} = G_{ij}^{\text{orig}} \cdot (1 - k \cdot d_{mt})(1 - k \cdot d_{mc}) \quad (5.17)$$

This approach neglects the damage evolution process and depends only on the load redistribution up till the failure effort reaches a value of one and generally predicts an unrealistically smooth degradation process [24].

5.2.4 COMPUTATIONAL PROCESS

A user defined material (UMAT) subroutine was developed for implementing the model and is used in conjunction with the commercial finite element software, ABAQUS.

Figure 1 shows the algorithm followed by the subroutine.

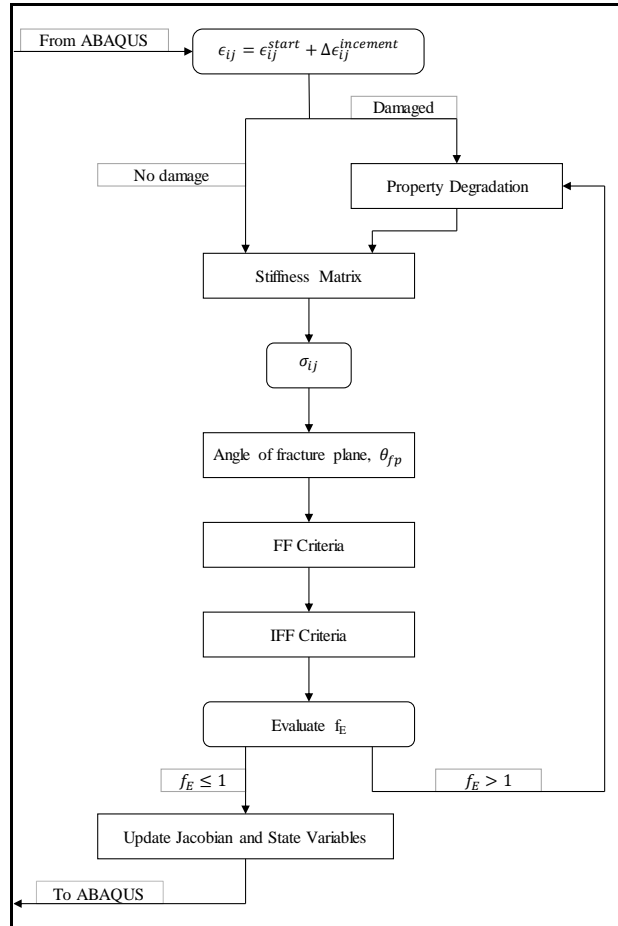


Figure 5.1 Algorithm for ABAQUS subroutine

ABAQUS calls the subroutine at the start of the increment and provides the initial and incremental strain along with the material properties. Using the constitutive equations, the stiffness matrix and the stress is calculated. This is then used as input parameters for the Puck failure criteria to detect damage in the material. If the stress exposure values reach or exceed

one, property degradation is carried out at the point until $f_E \leq 1$ or the maximum degradation has been reached. This process is carried out at every integration point in the material for each increment.

5.3. ANALYSIS

Progressive failure analysis of rectangular laminates with a through thickness hole loaded under in-plane tension was carried out. Four different dimensions were considered for the analysis as listed in Table 5.2. The analysis was implemented for three different layups – $[0/(\pm 45)_3/90_3]$, $[0/(\pm 45)_2/90_5]$, $[0/(\pm 45)_1/90_7]$ for each of the dimensions listed for a total of 12 cases. The material is T300/1034C carbon/epoxy laminate and the properties are listed in Table 5.3, and were taken from the paper published by Miami et al [29]. The longitudinal modulus of the fiber, $E_{\parallel f}$ was obtained from the datasheet of the material [25]. The volume fraction for the material was taken from [26].

The laminates were modeled using the composite layup option available in ABAQUS with 3 integration points through the thickness of each layer. A quarter of the model was simulated due to the symmetry of the balanced laminate. Symmetry boundary conditions were employed along the x and y symmetry planes. The geometry of the employed model can be seen in figure 2. An equation constraint interaction was employed at the loading surface with displacement controlled loading being applied at a reference node so that the displacements of the loading surface would be uniform across the nodes on that surface. This is done so that the reaction force and displacement of the surface can be obtained at the reference node. The laminate was modelled using C3D8R solid elements.

Table 5.2: Material Properties of T300/1034C

E_1 (GPa)	146.8
E_2 (GPa)	11.4
G_{12} (GPa)	6.1
V_{12}	0.3
V_{23}	0.3
E_{F1} (GPa)	230
V_{F12}	0.2
X_T (MPa)	1730
X_C (MPa)	1379
Y_T (MPa)	66.5
Y_C (MPa)	268.2
S_{12} (MPa)	58.7

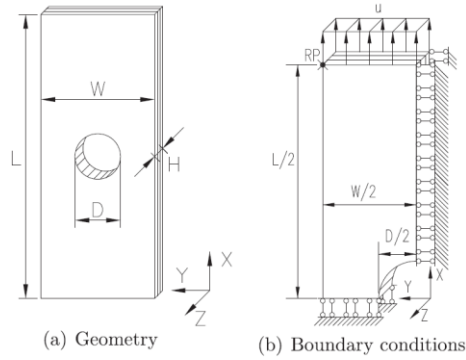


Figure 5.2 Geometry and Boundary conditions of the laminate from [26].

Table 5.3: Dimensions of the composites

Label	Length (mm)	Height (mm)	Width (mm)	Diameter (mm)
A	203.2	2.616	19.05	3.175
B	203.2	2.616	38.1	6.35
C	203.2	2.616	12.7	3.175
D	203.2	2.616	25.4	6.35

5.3.1 RESULTS

The results of the finite element analysis are shown in table 3. The failure stress in the model is obtained by using the formula, $\sigma_u = P_u / (W * H)$, where P_u is the load at failure and W and H are the width and the height of the specimen respectively and were compared with

the results taken from [26] wherein a combined elastoplastic model is tested for the same cases against predictions by Chang and Chang[27], Tan[28], Miami[29] and the experimental data from Chang et al[30].

Table 5.4: Comparison of Failure Stress and Percentage Error

Layup	Label	Failure Stress (MPa)						Error (%)				
		Present	Chen	Chang	Tan	Miami	Test Data	Present	Chen	Chang	Tan	Miami
$[0/(\pm 45)_3/90_3]_s$	A	271.69	293.07	227.53	275.75	-	277.17	-1.97	5.74	-17.91	-0.5	-
$[0/(\pm 45)_3/90_3]_s$	B	250.42	252.22	206.84	275.79	-	256.48	-2.36	-1.66	-19.35	7.53	-
$[0/(\pm 45)_3/90_3]_s$	C	227.79	269.05	206.84	262.00	-	226.15	0.72	18.97	-8.54	15.85	-
$[0/(\pm 45)_3/90_3]_s$	D	232.21	238.30	179.26	248.21	263.1	235.8	-1.52	1.06	-23.98	5.26	11.6
$[0/(\pm 45)_2/90_5]_s$	A	220.32	239.13	193.05	186.16	-	236.49	-6.8	1.12	-18.37	-21.28	-
$[0/(\pm 45)_2/90_5]_s$	B	200.96	214.30	172.37	186.16	-	204.08	-1.52	5.00	-15.54	-8.78	-
$[0/(\pm 45)_2/90_5]_s$	C	187.97	216.28	165.47	172.37	-	177.88	5.67	21.58	-6.98	-3.10	-
$[0/(\pm 45)_2/90_5]_s$	D	193.23	205.83	151.68	158.58	200.1	185.47	4.18	10.98	-18.22	-14.50	7.7
$[0/(\pm 45)_3/90_7]_s$	A	166.35	171.03	144.79	227.53	-	190.98	-12.89	-10.45	-24.19	19.13	-
$[0/(\pm 45)_3/90_7]_s$	B	148.52	150.36	124.11	227.53	-	158.58	-6.34	-5.18	-21.74	43.48	-
$[0/(\pm 45)_3/90_7]_s$	C	140.68	154.96	124.11	213.74	-	134.45	4.63	15.25	-7.69	58.97	-
$[0/(\pm 45)_3/90_7]_s$	D	146.7	135.67	103.42	199.95	148.2	159.96	-8.28	15.19	-35.34	25.00	-7.4

The Chen model has a high error percentage in models with layup dimensions C. It was found that layup C which is the smallest laminate in the tests requires a smaller mesh size than the other models to give satisfactory results.

The progressive damage buildup in the individual laminates is displayed for the $[0/(\pm 45)_1/90_7]_s$ laminate for label C. Figures 4,5 and 6 display the damage progression in the

0,45 and 90 degree plies of the laminate respectively. The damage first initiates in the 90 ply with IFF due to tensile forces. Damage then initiates in the 45 plies as it continues to accumulate in the 90 ply. The matrix damage in both these plies primarily follows the fiber direction. Finally, fiber failure due to tension occurs in the 0 ply and this propagates perpendicular to the zero degree fibers. This failure causes very high stresses on the neighboring plies and causes all plies to fail in the same direction as the 0 plies even though damage previously occurs and was propagating along the fiber directions in the 45 degree ply. Final failure occurs when damage propagates across the width for each of the plies.

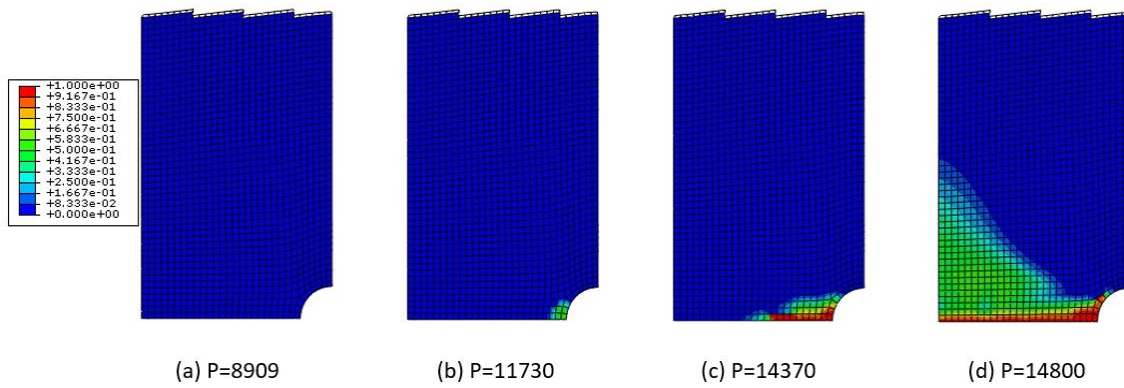


Figure 5.3 Damage propagation in 0 degree layer

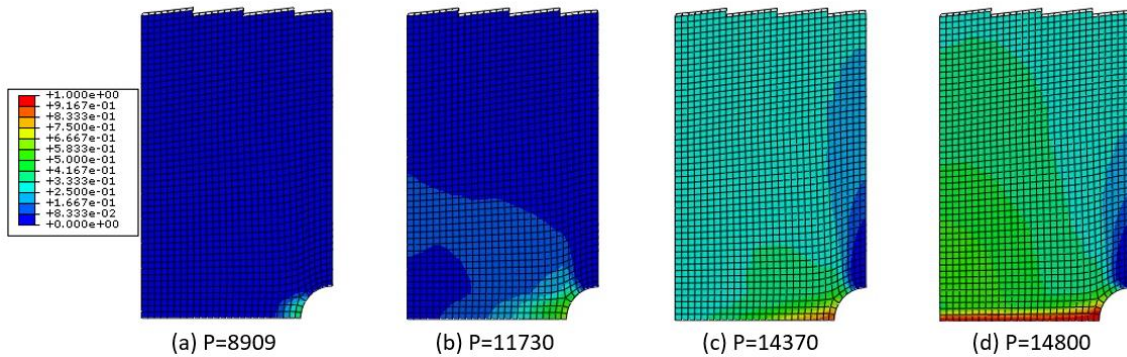


Figure 5.4 Damage propagation in 45 degree layer

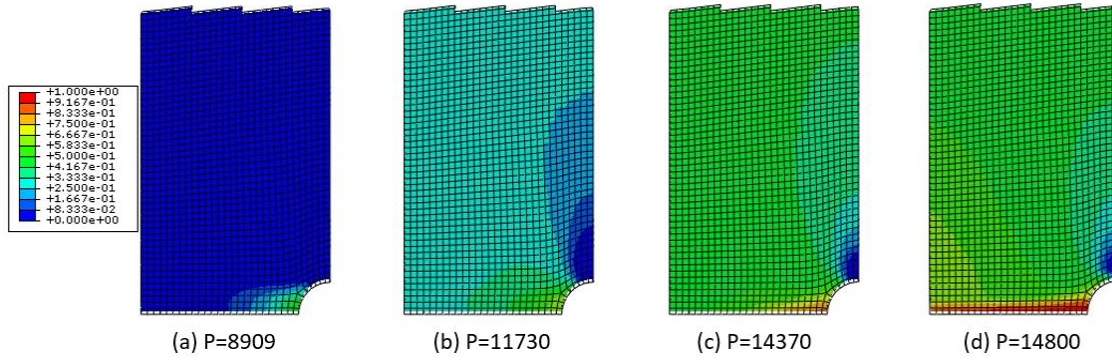


Figure 5.5 Damage propagation in the 90 degree layer

5.4 CONCLUSION

In the current work, the Puck failure criteria is successfully enforced to simulate the progressive failure of composite laminates under uniaxial tension. The results are in good agreement with the experimental data. The Puck criteria is a robust criterion capable of evaluating the progressive failure and does not require the calibration of any of the parameters. This can be applied to larger structures of composite materials to accurately model the progressive failure. However, it should be noted that the analysis time is relatively higher as the Puck criteria requires to iteratively obtain the angle of the fracture plane at each integration point for each increment.

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CHAPTER 6

SUMMARY AND RECOMMENDATIONS

6.1 SUMMARY

In the present study, a user material subroutine (UMAT) is developed in the FORTRAN environment for the commercially available finite element package ABAQUS. The developed code successfully estimates the progressive failure of the composite materials using the Puck failure criteria.

The following conclusions can be drawn from the results of this work:

- The failure stresses can be accurately predicted for composite laminates.
- The Puck failure criteria is a powerful failure criteria that can predict matrix damage in the material based on a physically sound hypothesis, adapted from Mohr's hypothesis.
- The model when coupled with a degradation model can be used to accurately predict the post failure degradation behavior.
- The Constant Stress Exposure (CSE) method is found to be better than the Element Weakening Method at evaluating the response of a material. However, it is also computationally more expensive.
- The proposed model can be applied to perform the failure analysis for larger, complex composite structures

- Only the material properties and material strengths are required to apply this failure criteria.

6.2 RECOMMENDATIONS

The following recommendations can be made with regards to future work to be conducted:

- The Puck criteria is computationally much more expensive than most other failure criteria as it requires a sequential computation for the angle of the fracture plane at each point. A more efficient method can be developed to find the angle of the fracture plane with fewer iterations.
- The model is highly mesh dependent, thus a process to reduce the mesh dependency would help to increase the efficiency of the model.
- A proper degradation model due to the FF must be set up. Currently FF is regarded as the final failure of the lamina in the Puck failure theory and little work has been done on the degradation laws for these damage modes.

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APPENDIX A

FORTRAN CODE

This code is available at CDMhub.com under Puck failure criteria along with a user manual.

```

C      Written by Karan Kodagali @ University of South Carolina at Columbia
C      Based largely on the theories of Alfred Puck
C      Please send any bugs or errors to kodagali@email.sc.edu
SUBROUTINE UMAT(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
1  RPL,DDSDDT,DRPLDE,DRPLDT,
2  STRAN,DSTRAN,TIME,DTIME,TEMP,DTEMP,PREDEF,DPRED,CMNAME,
3  NDI,NSHR,NTENS,NSTATEV,PROPS,NPROPS,COORDS,DROT,PNEWDT,
4  CELENT,DFGRD0,DFGRD1,NOEL,NPT,LAYER,KSPT,KSTEP,KINC)
C
C      INCLUDE 'ABA_PARAM.INC'
C
C      INTEGER
1  I,J
C
C      CHARACTER*80 CMNAME
C      CHARACTER*80 CPNAME
C      DIMENSION STRESS(NTENS),STATEV(NSTATEV),
1  DDSDDE(NTENS,NTENS),DDSDDT(NTENS),DRPLDE(NTENS),
2  STRAN(NTENS),DSTRAN(NTENS),TIME(2),PREDEF(1),DPRED(1),
3  PROPS(NPROPS),COORDS(3),DROT(3,3),DFGRD0(3,3),DFGRD1(3,3)
C      DOUBLE PRECISION
1  E1,E2,E3,G12,G13,G23,V12,V13,V23,V21,V31,V32,S(NTENS),
2  XT,XC,YT,YC,VF12,EF1,C0,S0,TMAT(6,6),STRA(NTENS),
3  C(NTENS,NTENS),S21,FFT,FFC,MFT,MFC,NMP,NW1,G230,
4  MFLC,DFT,DFC,DMT,DMC,DMLC,STRANT(NTENS),CD(NTENS,NTENS),
5  T21C,RVVA,ZERO,ONE,CF(NTENS,NTENS),MAXIM,E20,G120,
6  CFULL(NTENS,NTENS),MFMV,DMG,COUNTER,XX,VAR,E10,XTF,
7  RVVA1,YT1,S211,PTR,SIGN,TAUNT,TAUNL
C      INTEGER DEG,MAT
C      PARAMETER (ZERO=0.D0, ONE=1.D0)
C      real(16), parameter :: PI_16 = 4 * atan (1.0_16)
C
C      INITIALIZING VARIABLES-----
C
C      E1 = PROPS(1)           !YOUNG'S MODULUS IN DIRECTION 1 (L)
C      E2 = PROPS(2)           !YOUNG'S MODULUS IN DIRECTION 2 (T)
C      E3=E2
C      G12 = PROPS(3)          !SHEAR MODULUS IN 12 PLANE
C      G13=G12                 !SHEAR MODULUS IN 13 PLANE
C      V12=PROPS(4)            !POISSON RATIO IN 12
C      V23=PROPS(5)            !POISSON RATIO IN 23
C      V13=V12                 !POISSON RATIO IN 13
C      EF1=PROPS(6)            !MODULUS OF FIBER PARALLEL TO FIBER

```

```

VF12 = PROPS(7)          !VOLUME FRACTION OF FIBER
XT = PROPS(8)            !TENSILE STRENGTH PARALLEL TO FIBER
XC = PROPS(9)            !COMPRESSIVE STRENGTH PARALLEL TO FIBER
YT = PROPS(10)           !TENSILE STRENGTH PERPENDICULAR TO FIBER
YC = PROPS(11)           !COMPRESSIVE STRENGTH PERPENDICULAR TO FIBER
S21 = PROPS(12)          !IN PLANE SHEAR STRENGTH
MAT = PROPS(13)           !MATERIAL TYPE FOR INCLINATION PARAMETERS
DEG = PROPS(14)           !EWM/CSE
G23 = E2/2/(1.+V23)       !SHEAR MODULUS IN 23 PLANE
XTF=XT*EF1/E1            !EFFECTIVE TENSILE STRENGTH OF FIBER
C   ROT = PROPS(15)
C   C0=COS(ROT*PI_16/180)
C   S0=SIN(ROT*PI_16/180)
C   Damage variables from previous time step
DFT=STATEV(6)
DFC=STATEV(7)
DMT=STATEV(8)
DMC=STATEV(9)
C   Saving original stiffness values before degradation
E20=E2
G120=G12
G230=G23
E10=E1
C
DMG=ONE
V21=(E2/E1)*V12
V31=(E3/E1)*V13
V32=(E3/E2)*V23
C
C   STRAIN-----
C
DO I = 1, NTENS
    STRANT(I) = STRAN(I) + DSTRAN(I)
END DO
C
C   DEGRADATION DUE TO PREVIOUS DAMAGE-----
---
C
IF(DEG.EQ.1) THEN
    IF(MAT.EQ.1) DEGK=0.7
    IF(MAT.EQ.2) DEGK=0.41
    E1=(1-DFC)*(1-DFT)*E10
    G12=(1-DFT)*(1-(DMC*DEGK))*(1-(DMT*DEGK))*G120
    E2=(1-DFT)*(1-DMT)*E20
    E3=E2
    G13=G12
    G23=(1-DFT)*(1-(DMC*DEGK))*(1-(DMT*DEGK))*G230
    V21=(E2/E1)*V12
    V31=(E3/E1)*V13
    V32=(E3/E2)*V23
END IF
C
C   CONSTITUTIVE RESPONSE AND STRESS-----
-
C
CALL CONSTITUTIVE(CF,E1,E2,E3,G12,G13,G23,V12,V13,V23,V21,
1 V31,V32,NTENS,NDI,NSHR,DFT,DFC,DMT,DMC,DEG)
DO K1=1,NTENS

```

```

        S(K1)=0.0D0
        DO K2=1,NTENS
            S(K1)=S(K1)+CF(K2,K1)*STRANT(K2)
        ENDDO
    ENDDO
    STRESS=S
    DDSDE=CF      !Updating jacobian and stress is carried out with degradation at
the previous load step to ensure easier convergence
C                !Thus a small time step is required for accurate results.
C    DAMAGE EVALUATION AND DEGRADATION-----
--
C
    DO WHILE ( DMG.EQ.ONE )
        CALL CONSTITUTIVE(CF,E1,E2,E3,G12,G13,G23,V12,V13,V23,V21,
1      V31,V32,NTENS,NDI,NSHR,DFT,DFC,DMT,DMC,DEG)
        DO K1=1,NTENS
            S(K1)=0.0D0
            DO K2=1,NTENS
                S(K1)=S(K1)+CF(K2,K1)*STRANT(K2)
            ENDDO
        ENDDO
        CALL THETAFF(S,S21,XTF,XC,YT,YC,THETA,NTENS,NMP,MAXIM,NDI,
1      NSHR,MAT)
        CALL CFAILURE(S,V12,VF12,E1,EF1,S21,XTF,XC,YT,YC,NDI,NSHR,
1      MFT,FFT,FFC,MFC,DMG,NTENS,THETA,NMP,NW1,MAXIM,SIGN,MAT,TAUNT,
2      TAUNL,PTR,RVVA1,YT1,S211)
        IF(DFT.GT.0.99) FFT=1
        IF(DFC.GT.0.99) FFC=1
        IF(DMT.GE.0.99) MFT=1
        IF(DMC.GE.0.99) MFC=1
        IF(FFT.LE.ONE.AND.FFC.LE.ONE.AND.MFT.LE.ONE.AND.MFC.LE.
1      ONE) THEN
            DMG=ZERO
        END IF
C    Degradation due to damage in step for CSE
        IF(DEG.EQ.1) THEN
            IF(FFT.GT.ONE) THEN
                DFT=0.9999
            END IF
            IF(FFC.GT.ONE) THEN
                DFC=0.9999
            END IF
            IF(MFT.GT.ONE) THEN
                DMT=DMT+ONE/100
                IF(DMT.GE.ONE) THEN
                    DMT=0.99
                END IF
            END IF
            IF(MFC.GT.ONE) THEN
                DMC=DMC+ONE/100
                IF(DMC.GE.ONE) THEN
                    DMC=0.99
                END IF
            END IF
            E1=(1-DFC)*(1-DFT)*E10
            G12=(1-DFT)*(1-(DMC*0.7))*(1-(DMT*0.7))*G120
            E2=(1-DFT)*(1-DMT)*E20
            E3=E2

```

```

        G13=G12
        G23=(1-DFT)*(1-(DMC*0.7))*(1-(DMT*0.7))*G230
        V21=(E2/E1)*V12
        V31=(E3/E1)*V13
        V32=(E3/E2)*V23
C      Degradation due to damage in step for ewm
      ELSE IF(DEG.EQ.2) THEN
        IF(FFT.GT.ONE) THEN
          DFT=0.9999
        END IF
        IF(FFC.GT.ONE) THEN
          DFC=0.9999
        END IF
        IF(MFT.GT.ONE.AND.DMT.EQ.ZERO) THEN
          DMT=0.99
        END IF
        IF(MFC.GT.ONE.AND.DMC.EQ.ZERO) THEN
          DMC=0.99
        END IF
      END IF
    END DO
    THETA=THETA*180/(4 * atan (1.0_16))
C
C      SAVE STATE VARIABLES-----
C
    STATEV(1) = FFT
    STATEV(2) = FFC
    STATEV(3) = MFT
    STATEV(4) = MFC
    STATEV(5) = THETA
    STATEV(6) = DFT
    STATEV(7) = DFC
    STATEV(8) = DMT
    STATEV(9) = DMC
    STATEV(10) = MAX(DFT,DFC,DMT,DMC)
    RETURN
  END
C*****
C CALCULATE THE CONSTITUTIVE RESPONSE*****
C*****
  SUBROUTINE CONSTITUTIVE(CF,E1,E2,E3,G12,G13,G23,V12,V13,V23,V21,
1 V31,V32,NTENS,NDI,NSHR,DFT,DFC,DMT,DMC,DEG)
    INCLUDE 'ABA_PARAM.INC'
    DOUBLE PRECISION
2 E1,E2,G12,G23,V12,V13,V23,DMG,G13,V21,V31,V32,E3,DF,DM,DMT,DMC,
2 CF(NTENS,NTENS),S(NTENS),STRANT(6),ATEMP,DELTA,SMT,SMC,DFT,DFC
    INTEGER NDI,NTENS,DEG,NSHR
    PARAMETER (ZERO=0.D0, ONE=1.D0)
    DO K1=1,NTENS
      DO K2=1,NTENS
        CF(K1,K2)=0.D0
      ENDDO
    ENDDO
C      CONSTITUTIVE RESPONSE CALCULATED FOR 3D AND 2D CASES WITH CSE AND EWM
    DEGRADATION
      IF(NDI.EQ.3) THEN
        IF(DEG.EQ.1) THEN
          DELTA=1/(1-V12*V21-V23*V32-V13*V31-2*V21*V32*V13)

```

```

CF(1,1) = E1*(1-V23*V32)*DELTA
CF(1,2) = E2*(V12+V32*V13)*DELTA
CF(1,3) = E1*(V31+V21*V32)*DELTA
CF(2,1) = CF(1,2)
CF(2,2) = E2*(1-V13*V31)*DELTA
CF(2,3) = E2*(V32+V12*V31)*DELTA
CF(3,1) = CF(1,3)
CF(3,2) = CF(2,3)
CF(3,3) = E3*(1-V12*V21)*DELTA
CF(4,4) = G12
CF(5,5) = G13
CF(6,6) = G23
ELSE
SMT=0.9
SMC=0.5
DF=1-(1-DFT)*(1-DFC)
DM=1-(1-DMT)*(1-DMMC)
DELTA=1/(1-V12*V21-V23*V32-V13*V31-2*V21*V32*V13)
CF(1,1) = (1-DF)*E1*(1-V23*V32)*DELTA
CF(1,2) = (1-DF)*(1-DM)*E1*(V21+V31*V23)*DELTA
CF(1,3) = (1-DF)*(1-DM)*E1*(V31+V21*V32)*DELTA
CF(2,1) = CF(1,2)
CF(2,2) = (1-DF)*(1-DM)*E2*(1-V13*V31)*DELTA
CF(2,3) = (1-DF)*(1-DM)*E2*(V32+V12*V31)*DELTA
CF(3,1) = CF(1,3)
CF(3,2) = CF(2,3)
CF(3,3) = (1-DF)*(1-DM)*E3*(1-V12*V21)*DELTA
CF(4,4) = (1-DF)*(1-SMT*DMT)*(1-SMC*DMC)*G12
CF(5,5) = (1-DF)*(1-SMT*DMT)*(1-SMC*DMC)*G13
CF(6,6) = (1-DF)*(1-SMT*DMT)*(1-SMC*DMC)*G23
END IF
ELSE IF(NDI.EQ.2) THEN
IF(DEG.EQ.1) THEN
DELTA = 1-V12*V21
CF(1,1) = E1/DELTA
CF(2,2) = E2/DELTA
CF(1,2) = V12*E2/DELTA
CF(2,1) = CF(1,2)
CF(3,3) = G12
IF(NSHR.GT.1) THEN
CF(4,4) = G13
CF(5,5) = G23
END IF
ELSE
DELTA = 1-V12*V21
CF(1,1) = (1-DF)*E1/DELTA
CF(2,2) = (1-DF)*(1-DM)*E2/DELTA
CF(1,2) = (1-DF)*(1-DM)*V12*E2/DELTA
CF(2,1) = (1-DF)*(1-DM)*CF(1,2)
CF(3,3) = (1-DF)*(1-SMT*DMT)*(1-SMC*DMC)*G12
IF(NSHR.GT.1) THEN
CF(4,4) = (1-DF)*(1-SMT*DMT)*(1-SMC*DMC)*G13
CF(5,5) = (1-DF)*(1-SMT*DMT)*(1-SMC*DMC)*G23
END IF
END IF
END IF
RETURN
END

```

```

C*****
C ANGLE OF FRACTURE PLANE*****
C*****

SUBROUTINE THETAFF(S,S21,XT,XC,YT,YC,THETA,NTENS,NMP,MAXIM,NDI,
1 NSHR,MAT)
  INCLUDE 'ABA_PARAM.INC'
  INTEGER NTENS,NDI,NSHR,MAT
  DOUBLE PRECISION
1 P21T,P21C,P22C,S21,XT,XC,YT,YC,SIG13,SIG23,
2 S(NTENS),FE,MAXIM,MAXT,SIG11,SIG22,SIG33,SIG12,
3 RVVA,THETA,P22T,TAUNT,SIGN,TAUNL,PTR,PCR,COS2PSI,
4 SIN2PSI
  real(16), parameter :: PI = 4 * atan (1.0_16)
  PARAMETER (ZERO=0.D0, ONE=1.D0)
  IF(MAT.EQ.1) THEN
    P21T = 0.3
    P21C = 0.25
    P22C = 0.2
    P22T=P22C
  ELSE
    P21T = 0.35
    P21C = 0.3
    P22C = 0.3
    P22T=P22C
  END IF
  IF(NDI.EQ.3) THEN
    SIG11=S(1)
    SIG22=S(2)
    SIG33=S(3)
    SIG12=S(4)/2
    SIG13=S(5)/2
    SIG23=S(6)/2
  ELSE
    SIG11=S(1)
    SIG22=S(2)
    SIG12=S(3)
    IF(NSHR.GT.1) THEN
      SIG33=0
      SIG13=S(4)/2
      SIG23=S(5)/2
    ELSE
      SIG33=0
      SIG13=0
      SIG23=0
    END IF
  END IF
  RVVA = (S21/(2*P21C))*(sqrt((1+2*P21C*YC/S21))-1)
  DO I = -90,90
    THETA = I*PI/180
    SIGN=SIG22*(COS(THETA))**2+SIG33*(SIN(THETA))**2+2*SIG23*
1 SIN(THETA)*COS(THETA)
    TAUNT=-SIG22*SIN(THETA)*COS(THETA)+SIG33*SIN(THETA)*COS(THETA)
1 +SIG23*((COS(THETA))**2-(SIN(THETA))**2)
    TAUNL=SIG13*SIN(THETA)+SIG12*COS(THETA)
    COS2PSI=TAUNT**2/(TAUNT**2+TAUNL**2)
    SIN2PSI=TAUNL**2/(TAUNT**2+TAUNL**2)
    PTR=(P22T/RVVA)*COS2PSI+(P21T/S21)*SIN2PSI
    PCR=(P22C/RVVA)*COS2PSI+(P21C/S21)*SIN2PSI
  
```

```

        IF(SIGN.GE.ZERO) THEN
            FE =SQRT((((1/YT)-PTR)*SIGN)**2+(TAUNT/RVVA)**2+
1          (TAUNL/S21)**2)+PTR*SIGN
        ELSE
            FE = SQRT((TAUNT/RVVA)**2+(TAUNL/S21)**2+(PCR*SIGN)**2)+
1          PCR*SIGN
        END IF
        IF(FE.GT.MAXIM) THEN
            MAXIM=FE
            MAXT=THETA
        END IF
    END DO
    THETA=MAXT          ! Angle of Fracture Plane
    RETURN
END

C*****
C PUCK FAILURE CRITERIA*****
C*****

SUBROUTINE CFAILURE(S,V12,VF12,E1,EF1,S211,XT,XC,YT1,YC1,NDI,NSHR,
1 MFT,FFT,FFC,MFC,DMG,NTENS,THETA,NMP,NW1,MAXIM,SIGN,MAT,TAUNT,
2 TAUNL,PTR,RVVA,YT,S21)
    INCLUDE 'ABA_PARAM.INC'
    INTEGER NTENS,KINC,NOEL
    DOUBLE PRECISION
1 P21T,P21C,P22C,MSIG,VF12,EF1,S21,XT,XC,YT,YC,MAXIM,E1,
2 S(NTENS),STRANT(NTENS),MFT,FFT,FFC,MFC,DMG,V12,SIG23,
3 T21C,RVVA,THETA,P22T,TAUNT,SIGN,TAUNL,PTR,PCR,COS2PSI,SIG13,
4 SIN2PSI,NMP,M,SC,A,C,NW1,YT1,YC1,S211,SIG11,SIG22,SIG33,SIG12
    PARAMETER (ZERO=0.D0, ONE=1.D0)
    IF(NDI.EQ.3) THEN
        SIG11=S(1)
        SIG22=S(2)
        SIG33=S(3)
        SIG12=S(4)/2
        SIG13=S(5)/2
        SIG23=S(6)/2
    ELSE
        SIG11=S(1)
        SIG22=S(2)
        SIG12=S(3)
        IF(NSHR.GT.1) THEN
            SIG33=0
            SIG13=S(4)/2
            SIG23=S(5)/2
        ELSE
            SIG33=0
            SIG13=0
            SIG23=0
        END IF
    END IF
    IF(MAT.EQ.1) THEN
        P21T = 0.3
        P21C = 0.25
        P22C = 0.2
        P22T = P22C
        MSIG = 1.3
    ELSE
        P21T = 0.35

```

```

        P21C = 0.3
        P22C = 0.25
        P22T = P22C
        MSIG = 1.1
    END IF
C    RVVA = YC/(2*(1+P22C))
C
C    FAILURE CRITERIA
C
!    FIBER TENSILE
    IF((SIG11-(V12-VF12*MSIG*E1/EF1)*(SIG22+SIG33)).GE.ZERO) THEN
        FFT = (SIG11-(V12-VF12*MSIG*E1/EF1)*(SIG22+SIG33))/XT
        FFC = ZERO
        IF(FFT.GE.ONE) THEN
            DMG=ONE
        END IF
    ELSE
!    FIBER COMPRESSIVE
        FFC = ABS((SIG11-(V12-VF12*MSIG*E1/EF1)*(SIG22+SIG33)))/XC
        FFT = ZERO
        IF(FFC.GE.ONE) THEN
            DMG=ONE
        END IF
    END IF
    YT=YT1
    YC=YC1
    S21=S211
    RVVA = (S21/(2*P21C))*(sqrt((1+2*P21C*YC/S21))-1)
    SIGN=SIG22*(COS(THETA)**2+SIG33*(SIN(THETA))**2+2*SIG23*SIN(THETA
1) *COS(THETA)
    TAUNT=-SIG22*SIN(THETA)*COS(THETA)+SIG33*SIN(THETA)*COS(THETA)+
1 SIG23*((COS(THETA))**2-(SIN(THETA))**2)
    TAUNL=SIG13*SIN(THETA)+SIG12*COS(THETA)
    COS2PSI=TAUNT**2/(TAUNT**2+TAUNL**2)
    SIN2PSI=TAUNL**2/(TAUNT**2+TAUNL**2)
    PTR=(P22T/RVVA)*COS2PSI+(P21T/S21)*SIN2PSI
    PCR=(P22C/RVVA)*COS2PSI+(P21C/S21)*SIN2PSI
!    MATRIX TENSILE
    IF(SIGN.GE.ZERO) THEN
        MFT =SQRT((((1/YT)-PTR)*SIGN)**2+(TAUNT/RVVA)**2+(TAUNL/S21)
1 **2)+PTR*SIGN
        MFC=ZERO
    END IF
!    MATRIX COMPRESSION
    IF(SIGN.LT.ZERO) THEN
        MFC = SQRT((TAUNT/RVVA)**2+(TAUNL/S21)**2+(PCR*SIGN)**2)+PCR*SIGN
        MFT=ZERO
    END IF
    IF(MFC.GE.ONE.OR.MFT.GE.ONE.OR.FFT.GE.ONE.OR.FFC.GE.ONE) THEN
        DMG=ONE
    ELSE
        DMG=ZERO
    END IF
    RETURN
END

```

