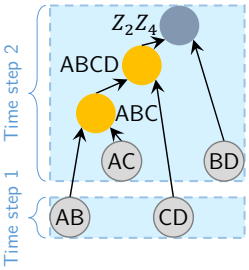
Modicum protocol

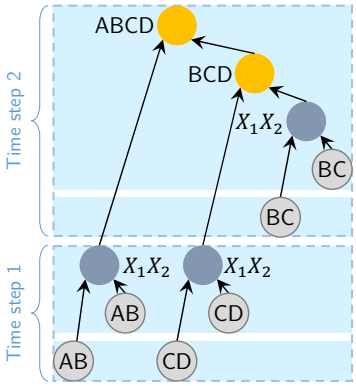


Time step 1:
Time block AB:
CREATE LINK between qubits [A, 1] and [B, 1].
SWAP qubits [A, 1] ↔ [A, 2] and qubits [B, 1] ↔ [B, 2].
Time block CD:
CREATE LINK between qubits [C, 1] and [D, 1].
SWAP qubits [C, 1] ↔ [C, 2] and qubits [D, 1] ↔ [D, 2].

Time step 2:
Time block AC:
CREATE LINK between qubits [A, 1] and [C, 1].
(R1) *FUSE* by measuring qubits [A, 1] and keeping qubits [A, 2].
(R2) *FUSE* by measuring qubits [C, 1] and keeping qubits [C, 2].
Time block BD:
CREATE LINK between qubits [B, 1] and [D, 1].
(R3) *DISTILL* operation Z_2Z_4 by measuring qubits [B, 1] and [D, 1], and keeping qubits [B, 2] and [D, 2].

EVALUATE the success of R3 based on R1⊕R2⊕R3.
CORRECT qubit [C, 2] with operator X conditioned on R1⊕R2.
CORRECT qubit [D, 2] with operator X conditioned on R1⊕R2.

Sextimum protocol

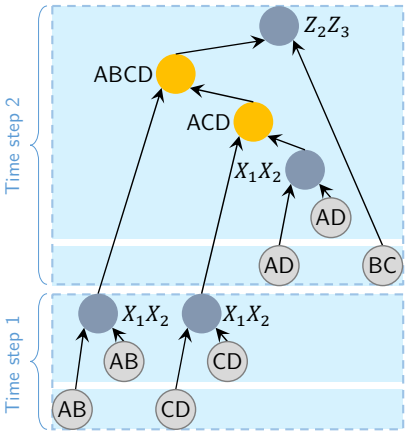


Time step 1:
Time block AB (2 entanglement links):
CREATE LINK between qubits [A, 1] and [B, 1].
SWAP qubits [A, 1] ↔ [A, 2] and qubits [B, 1] ↔ [B, 2].
CREATE LINK between qubits [A, 1] and [B, 1].
DISTILL operation X_1X_2 by measuring qubits [A, 1] and [B, 1], and keeping qubits [A, 2] and [B, 2].
Time block CD (2 entanglement links):
CREATE LINK between qubits [C, 1] and [D, 1].
SWAP qubits [C, 1] ↔ [C, 2] and qubits [D, 1] ↔ [D, 2].
CREATE LINK between qubits [C, 1] and [D, 1].
DISTILL operation X_1X_2 by measuring qubits [C, 1] and [D, 1], and keeping qubits [C, 2] and [D, 2].

Time step 2:
Time block BC (2 entanglement links):
CREATE LINK between qubits [B, 1] and [C, 1].
SWAP qubits [B, 1] ↔ [B, 3] and qubits [C, 1] ↔ [C, 3].
CREATE LINK between qubits [B, 1] and [C, 1].
DISTILL operation X_1X_2 by measuring qubits [B, 1] and [C, 1], and keeping qubits [B, 3] and [C, 3].
SWAP qubits [B, 1] ↔ [B, 3] and qubits [C, 1] ↔ [C, 3].
(R1) *FUSE* by measuring qubits [C, 1] and keeping qubits [C, 2].
(R2) *FUSE* by measuring qubits [B, 1] and keeping qubits [B, 2].

CORRECT qubit [C, 2] with operator X conditioned on R1⊕R2.
CORRECT qubit [D, 2] with operator X conditioned on R1⊕R2.

Septimum protocol

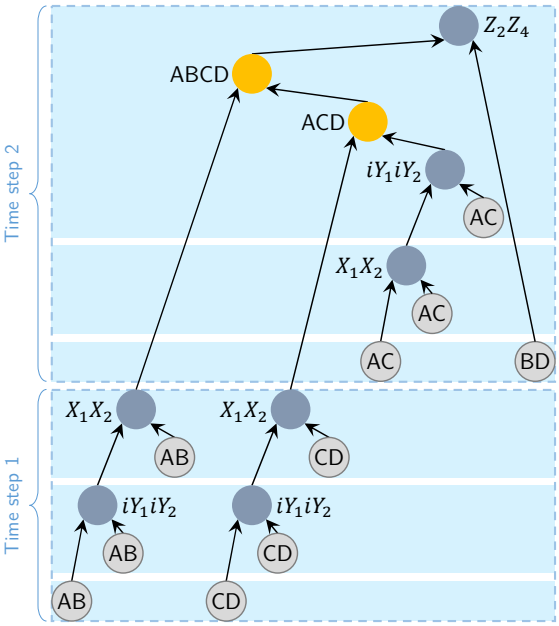


Time step 1:
Time block AB (2 entanglement links):
CREATE LINK between qubits [A, 1] and [B, 1].
SWAP qubits [A, 1] ↔ [A, 2] and qubits [B, 1] ↔ [B, 2].
CREATE LINK between qubits [A, 1] and [B, 1].
DISTILL operation X_1X_2 by measuring qubits [A, 1] and [B, 1], and keeping qubits [A, 2] and [B, 2].
Time block CD (2 entanglement links):
CREATE LINK between qubits [C, 1] and [D, 1].
SWAP qubits [C, 1] ↔ [C, 2] and qubits [D, 1] ↔ [D, 2].
CREATE LINK between qubits [C, 1] and [D, 1].
DISTILL operation X_1X_2 by measuring qubits [C, 1] and [D, 1], and keeping qubits [C, 2] and [D, 2].

Time step 2:
Time block AD (2 entanglement links):
CREATE LINK between qubits [A, 1] and [D, 1].
SWAP qubits [A, 1] ↔ [A, 3] and qubits [D, 1] ↔ [D, 3].
CREATE LINK between qubits [A, 1] and [D, 1].
DISTILL operation X_1X_2 by measuring qubits [A, 1] and [D, 1], and keeping qubits [A, 3] and [D, 3].
SWAP qubits [A, 1] ↔ [A, 3] and qubits [D, 1] ↔ [D, 3].
(R1) *FUSE* by measuring qubits [A, 1] and keeping qubits [A, 2].
(R2) *FUSE* by measuring qubits [D, 1] and keeping qubits [D, 2].
Time block BC (1 entanglement link):
CREATE LINK between qubits [B, 1] and [C, 1].
(R3) *DISTILL* operation Z_2Z_3 by measuring qubits [B, 1] and [C, 1], and keeping qubits [B, 2] and [C, 2].

EVALUATE the success of R3 based on R1⊕R2⊕R3.
CORRECT qubit [C, 2] with operator X conditioned on R1⊕R2.
CORRECT qubit [D, 2] with operator X conditioned on R1⊕R2.

Decimum protocol

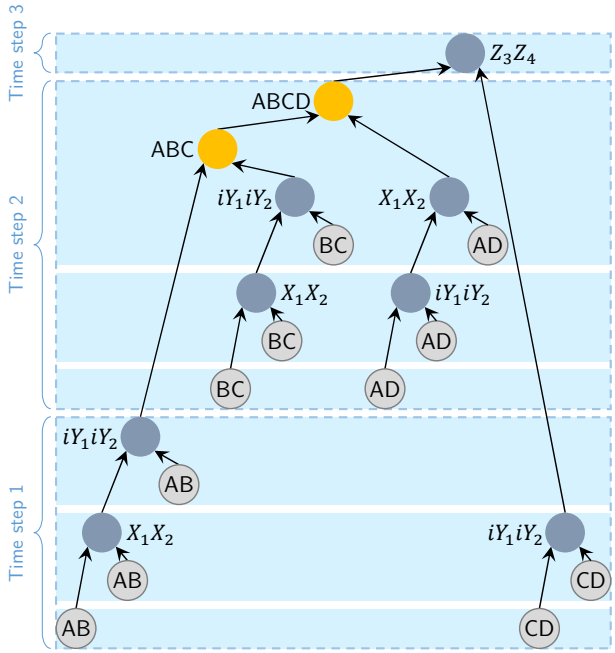


Time step 1:
Time block AB (3 entanglement links):
CREATE LINK between qubits [A, 1] and [B, 1].
SWAP qubits [A, 1] ↔ [A, 2] and qubits [B, 1] ↔ [B, 2].
CREATE LINK between qubits [A, 1] and [B, 1].
DISTILL operation iY_1iY_2 by measuring qubits [A, 1] and [B, 1], and keeping qubits [A, 2] and [B, 2].
CREATE LINK between qubits [A, 1] and [B, 1].
DISTILL operation X_1X_2 by measuring qubits [A, 1] and [B, 1], and keeping qubits [A, 2] and [B, 2].
Time block CD (3 entanglement links):
CREATE LINK between qubits [C, 1] and [D, 1].
SWAP qubits [C, 1] ↔ [C, 2] and qubits [D, 1] ↔ [D, 2].
CREATE LINK between qubits [C, 1] and [D, 1].
DISTILL operation iY_1iY_2 by measuring qubits [C, 1] and [D, 1], and keeping qubits [C, 2] and [D, 2].
CREATE LINK between qubits [C, 1] and [D, 1].
DISTILL operation X_1X_2 by measuring qubits [C, 1] and [D, 1], and keeping qubits [C, 2] and [D, 2].

Time step 2:
Time block AC (3 entanglement links):
CREATE LINK between qubits [A, 1] and [C, 1].
SWAP qubits [A, 1] ↔ [A, 3] and qubits [C, 1] ↔ [C, 3].
CREATE LINK between qubits [A, 1] and [C, 1].
DISTILL operation X_1X_2 by measuring qubits [A, 1] and [C, 1], and keeping qubits [A, 3] and [C, 3].
CREATE LINK between qubits [A, 1] and [C, 1].
DISTILL operation iY_1iY_2 by measuring qubits [A, 1] and [C, 1], and keeping qubits [A, 3] and [C, 3].
SWAP qubits [A, 1] ↔ [A, 3] and qubits [C, 1] ↔ [C, 3].
(R1) *FUSE* by measuring qubits [A, 1] and keeping qubits [A, 2].
(R2) *FUSE* by measuring qubits [C, 1] and keeping qubits [C, 2].
Time block BD (1 entanglement link):
CREATE LINK between qubits [B, 1] and [D, 1].
(R3) *DISTILL* operation Z_2Z_4 by measuring qubits [B, 1] and [D, 1], and keeping qubits [B, 2] and [D, 2].

EVALUATE the success of R3 based on R1⊕R2⊕R3.
CORRECT qubit [C, 2] with operator X conditioned on R1⊕R2.
CORRECT qubit [D, 2] with operator X conditioned on R1⊕R2.

Undecum protocol

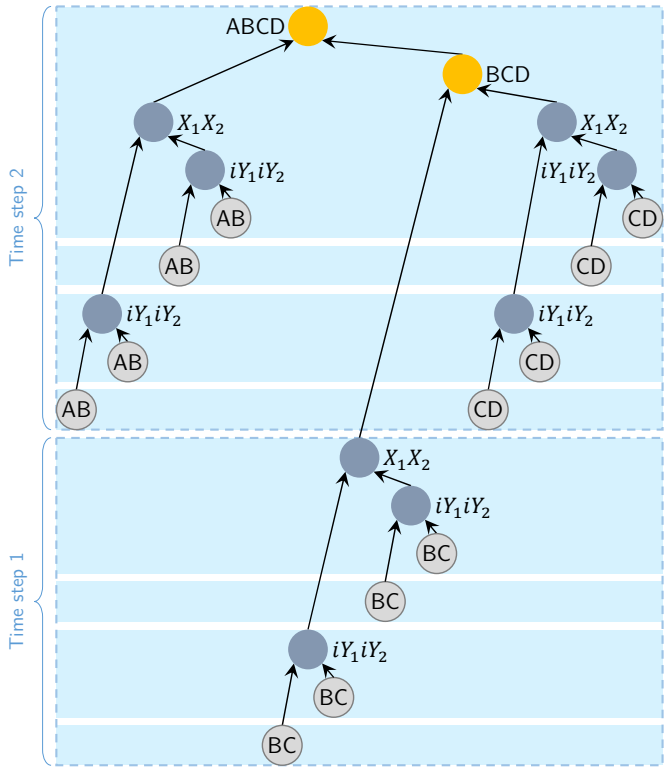


Time step 1:
Time block AB (3 entanglement links):
CREATE LINK between qubits [A, 1] and [B, 1].
SWAP qubits [A, 1] ↔ [A, 2] and qubits [B, 1] ↔ [B, 2].
CREATE LINK between qubits [A, 1] and [B, 1].
DISTILL operation X_1X_2 by measuring qubits [A, 1] and [B, 1], and keeping qubits [A, 2] and [B, 2].
CREATE LINK between qubits [A, 1] and [B, 1].
DISTILL operation iY_1iY_2 by measuring qubits [A, 1] and [B, 1], and keeping qubits [A, 2] and [B, 2].
Time block CD (2 entanglement links):
CREATE LINK between qubits [C, 1] and [D, 1].
SWAP qubits [C, 1] ↔ [C, 2] and qubits [D, 1] ↔ [D, 2].
CREATE LINK between qubits [C, 1] and [D, 1].
DISTILL operation iY_1iY_2 by measuring qubits [C, 1] and [D, 1], and keeping qubits [C, 2] and [D, 2].

Time step 2:
Time block BC (3 entanglement links):
CREATE LINK between qubits [B, 1] and [C, 1].
SWAP qubits [B, 1] ↔ [B, 3] and qubits [C, 1] ↔ [C, 3].
CREATE LINK between qubits [B, 1] and [C, 1].
DISTILL operation X_1X_2 by measuring qubits [B, 1] and [C, 1], and keeping qubits [B, 3] and [C, 3].
CREATE LINK between qubits [B, 1] and [C, 1].
DISTILL operation iY_1iY_2 by measuring qubits [B, 1] and [C, 1], and keeping qubits [B, 3] and [C, 3].
SWAP qubits [B, 1] ↔ [B, 3] and qubits [C, 1] ↔ [C, 2].
(R1) *FUSE* by measuring qubits [B, 1] and keeping qubits [B, 2].
CORRECT qubit [C, 3] with operator X conditioned on R1.
Time block AD (3 entanglement links):
CREATE LINK between qubits [A, 1] and [D, 1].
SWAP qubits [A, 1] ↔ [A, 3] and qubits [D, 1] ↔ [D, 3].
CREATE LINK between qubits [A, 1] and [D, 1].
DISTILL operation iY_1iY_2 by measuring qubits [A, 1] and [D, 1], and keeping qubits [A, 3] and [D, 3].
CREATE LINK between qubits [A, 1] and [D, 1].
DISTILL operation X_1X_2 by measuring qubits [B, 1] and [D, 1], and keeping qubits [A, 3] and [D, 3].
SWAP qubits [A, 1] ↔ [A, 3] and qubits [D, 1] ↔ [D, 2].
(R2) *FUSE* by measuring qubits [A, 1] and keeping qubits [A, 2].
CORRECT qubit [D, 3] with operator X conditioned on R2.

Time step 3:
Time block CD (0 entanglement links):
DISTILL operation Z_3Z_4 by measuring qubits [C, 1] and [D, 1], and keeping qubits [C, 3] and [D, 3].

Duodecum protocol



Time step 1:
Time block BC (4 entanglement links):
CREATE LINK between qubits [B, 1] and [C, 1].
SWAP qubits [B, 1] ↔ [B, 2] and qubits [C, 1] ↔ [C, 2].
CREATE LINK between qubits [B, 1] and [C, 1].
DISTILL operation iY_1iY_2 by measuring qubits [B, 1] and [C, 1], and keeping qubits [B, 2] and [C, 2].
CREATE LINK between qubits [B, 1] and [C, 1].
SWAP qubits [B, 1] ↔ [B, 3] and qubits [C, 1] ↔ [C, 3].
CREATE LINK between qubits [B, 1] and [C, 1].
DISTILL operation iY_1iY_2 by measuring qubits [B, 1] and [C, 1], and keeping qubits [B, 3] and [C, 3].
SWAP qubits [B, 1] ↔ [B, 3] and qubits [C, 1] ↔ [C, 3].
DISTILL operation X_1X_2 by measuring qubits [B, 1] and [C, 1], and keeping qubits [B, 2] and [C, 2].

Time step 2:
Time block AB (4 entanglement links):
CREATE LINK between qubits [A, 1] and [B, 1].
SWAP qubits [A, 1] ↔ [A, 2] and qubits [B, 1] ↔ [B, 3].
CREATE LINK between qubits [A, 1] and [B, 1].
DISTILL operation iY_1iY_2 by measuring qubits [A, 1] and [B, 1], and keeping qubits [A, 2] and [B, 3].
CREATE LINK between qubits [A, 1] and [B, 1].
SWAP qubits [A, 1] ↔ [A, 3] and qubits [B, 1] ↔ [B, 4].
CREATE LINK between qubits [A, 1] and [B, 1].
DISTILL operation iY_1iY_2 by measuring qubits [A, 1] and [B, 1], and keeping qubits [A, 3] and [B, 4].
SWAP qubits [A, 1] ↔ [A, 3] and qubits [B, 1] ↔ [B, 4].
DISTILL operation X_1X_2 by measuring qubits [A, 1] and [B, 1], and keeping qubits [A, 2] and [B, 3].
SWAP qubits [B, 1] ↔ [B, 2].
(R1) *FUSE* by measuring qubits [B, 1] and keeping qubits [B, 3].
Time block CD (4 entanglement links):
CREATE LINK between qubits [C, 1] and [D, 1].
SWAP qubits [C, 1] ↔ [C, 3] and qubits [D, 1] ↔ [D, 2].
CREATE LINK between qubits [C, 1] and [D, 1].
DISTILL operation iY_1iY_2 by measuring qubits [C, 1] and [D, 1], and keeping qubits [C, 3] and [D, 2].
CREATE LINK between qubits [C, 1] and [D, 1].
SWAP qubits [C, 1] ↔ [C, 4] and qubits [D, 1] ↔ [D, 3].
CREATE LINK between qubits [C, 1] and [D, 1].
DISTILL operation iY_1iY_2 by measuring qubits [C, 1] and [D, 1], and keeping qubits [C, 4] and [D, 3].
SWAP qubits [C, 1] ↔ [C, 4] and qubits [D, 1] ↔ [D, 3].
DISTILL operation X_1X_2 by measuring qubits [C, 1] and [D, 1], and keeping qubits [C, 3] and [D, 2].
SWAP qubits [C, 1] ↔ [C, 3].
(R2) *FUSE* by measuring qubits [C, 1] and keeping qubits [C, 2].

CORRECT qubit [C, 2] with operator X conditioned on R1.
CORRECT qubit [D, 2] with operator X conditioned on R1⊕R2.