Hysteretic benchmark with a dynamic nonlinearity

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1 Introduction

Hysteresis is a phenomenology commonly encountered in very diverse engineering and science disciplines, ranging from solid mechanics, electromagnetism and aerodynamics [1, 2, 3] to biology, ecology and psychology [4, 5, 6]. The defining property of a hysteretic system is the persistence of an input-output loop as the input frequency approaches zero [7]. Hysteretic systems are inherently nonlinear, as the quasi-static existence of a loop requires an input-output phase shift different from 0 and 180 degrees, which are the only two options offered by linear theory. The root cause of hysteresis is multistability [8]. A hysteretic system possesses multiple stable equilibria, attracting the output depending on the input history. In this sense, it is appropriate to refer hysteresis as system nonlinear memory.

This document describes the synthesis of noisy data exhibiting hysteresis behaviour carried out by combining the Bouc-Wen differential equations (Section 2) and the Newmark integration rules (Section 3). User guidelines to an accurate simulation are provided in Section 4. The test signals and the figures of merit that are used in this benchmark are presented in Section 5. Anticipated nonlinear system identification challenges associated with the present benchmark are listed in Section 6.

2 The Bouc-Wen model of hysteresis

The Bouc-Wen model [9, 10] has been intensively exploited during the last decades to represent hysteretic effects in mechanical engineering, especially in the case of random vibrations. Extensive literature reviews about phenomenological and applied aspects related to Bouc-Wen modelling can be found in Refs. [11, 12].

The vibrations of a single-degree-of-freedom Bouc-Wen system, *i.e.* a Bouc-Wen oscillator with a single mass, is governed by Newton's law of dynamics written in the form [10]

$$m_L\, \ddot{y}(t) + r(y, \dot{y}) + z(y, \dot{y}) = u(t), \tag{1}$$

where m_L is the mass constant, y the displacement, u the external force, and where an over-dot indicates a derivative with respect to the time variable t. The total restoring force in the system is composed of a static nonlinear term $r(y, \dot{y})$, which only depends on the instantaneous values of the displacement $y(t)$ and velocity $\dot{y}(t)$, and of a dynamic, *i.e.* history-dependent, nonlinear term $z(y, \dot{y})$, which encodes the hysteretic memory of the system. In the present study, the static restoring force contribution is assumed to be linear, that is

$$r(y, \dot{y}) = k_L\, y + c_L\, \dot{y}, \tag{2}$$

where k_L and c_L are the linear stiffness and viscous damping coefficients, respectively. The hysteretic force $z(y, \dot{y})$ obeys the first-order differential equation

$$\dot{z}(y, \dot{y}) = \alpha\, \dot{y} - \beta \left(\gamma\; |\dot{y}|\, |z|^{\nu-1}\, z + \delta\, \dot{y}\, |z|^{\nu}\right), \tag{3}$$

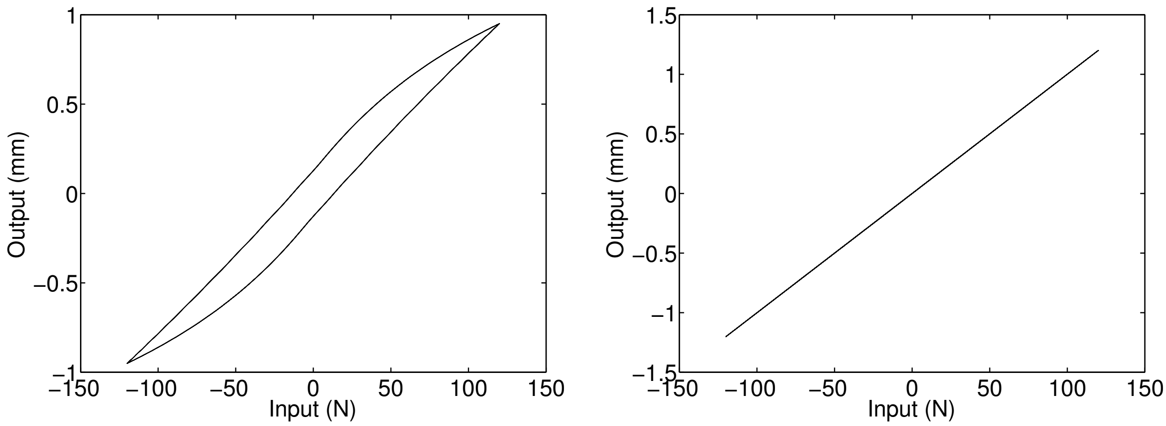
where the five Bouc-Wen parameters α, β, γ, δ and ν are used to tune the shape and the smoothness of the system hysteresis loop. Table 1 lists the values of the physical parameters selected in this study. The linear modal parameters deduced from m_L, c_L and k_L are given in Table 2. Fig. 1 (a) illustrates the existence of a non-degenerate loop in the system input-output plane for quasi-static forcing conditions. In comparison, by setting the β parameter to 0, a linear behaviour is retrieved in Fig. 1 (b). The excitation $u(t)$ in these two figures is a sine wave with a frequency of 1 Hz and an amplitude of 120 N. The response exhibits no initial condition transients as it is depicted over 10 cycles in steady state.

Parameter	m_L	c_L	k_L	α	β	γ	δ	ν
Value (in SI unit)	2	10	$5\,10^4$	$5\,10^4$	$1\,10^3$	0.8	-1.1	1

Table 1: Physical parameters of the Bouc-Wen system.

Parameter	Natural frequency ω_0 (Hz)	Damping ratio ζ (%)
Value	35.59	1.12

Table 2: Linear modal parameters of the Bouc-Wen system.



(a) (b)

Figure 1: Hysteresis loop in the system input-output plane for quasi-static forcing conditions. (a) Non-degenerate loop obtained for the parameters in Table 1; (b) linear behaviour retrieved when setting the β parameter to 0.

3 Time integration

The Bouc-Wen dynamics in Eqs. (1) and (3) can be effectively integrated in time using a Newmark method. Newmark integration relies on one-step-ahead approximations of the velocity and displacement fields obtained by applying Taylor expansion and numerical quadrature techniques [13]. Denoting by h the integration time step, these approximation relations write

$$\begin{aligned} \dot{y}(t+h) &= \dot{y}(t) + (1-a)\ h\,\ddot{y}(t) + a\,h\,\ddot{y}(t+h) \\ y(t+h) &= y(t) + h\,\dot{y}(t) + \left(\tfrac{1}{2}-b\right)\ h^2\,\ddot{y}(t) + b\,h^2\,\ddot{y}(t+h). \end{aligned} \tag{4}$$

Parameters a and b are typically set to 0.5 and 0.25, respectively. Eqs. (4) are herein enriched with an integration formula for the variable $z(t)$, which takes the form

$$z(t+h) = z(t) + (1-c)\ h\,\dot{z}(t) + c\,h\,\dot{z}(t+h), \tag{5}$$

where c, similarly to a, is set to 0.5. Based on Eqs. (4) and (5), a Newmark scheme proceeds in two steps. First, predictions of $\dot{y}(t+h)$, $y(t+h)$ and $z(t+h)$ are calculated assuming $\ddot{y}(t+h)=0$ and $\dot{z}(t+h)=0$. Second, the initial predictors are corrected via Newton-Raphson iterations so as to satisfy the dynamic equilibria in Eqs. (1) and (3).

4 User guidelines to an accurate simulation

The Newmark integration of the Bouc-Wen dynamics in Eqs. (1) and (3) is implemented in the Matlab encrypted p-file `BoucWen_NewmarkIntegration.p`. This function features 5 inputs, namely:

- the integration time step h;
- the external force time history $u(t)$;
- the initial value of the displacement $y(t=0)$;
- the initial value of the velocity $\dot{y}(t=0)$;
- the initial value of the hysteretic force $z(t=0)$.

The single output of the function is the displacement time history $y(t)$.

Based on the authors' experience with the Newmark integration of the Bouc-Wen system of Section 2, the following guidelines are formulated:

- it is suggested to consider a working sampling frequency of 750 Hz in order to properly observe the harmonic components generated by the nonlinearity;
- it is *strongly advised* to upsample the input force $u(t)$ by a factor 20 during time integration to guarantee the accuracy of the resulting displacement time series. This comes down to setting the integration sampling frequency, *i.e.* $1/h$, to 15000 Hz;
- after integration, the output sequence $y(t)$ may be low-pass filtered and downsampled using the Matlab command `decimate`. Note that this command belongs to the Matlab Signal Processing toolbox;
- low-pass filtering may be achieved using a 30-th order FIR filter (argument `'fir'` of the `decimate` command), paying attention to the inherent edge effects of the filter;
- the `decimate` command may be called several times breaking the downsampling argument, *e.g.* 20, into its prime factors, *e.g.* 2 - 2 - 5, to enhance numerical precision;
- initial conditions on $y(t)$, $\dot{y}(t)$ and $z(t)$ are usually set to 0.

The minimal working example file `BoucWen_ExampleIntegration.m` implements all these guidelines. In this example, a multisine excitation [14] is applied to the Bouc-Wen system considering all excited frequencies in the $5-150$ Hz band and a frequency resolution $f_0 = f_s/N \cong 0.09\, Hz$, given a sampling frequency $f_s = 750\, Hz$ and a number of time samples $N = 8192$. The root-mean-squared amplitude of the input is 50 N and 5 output periods are simulated. The sampling rate during integration is set to 15000 Hz. The synthesised displacement time history is low-pass filtered and downsampled back to 750 Hz.

In more details:

- the working and integration sampling frequencies are defined in section `Time integration parameters` on line 10;

- the excitation signal is designed in section `Excitation signal design` on line 17. Note that the Newmark simulation algorithm supports, in principle, all types of input time series;
- initial conditions are set on lines 43, 44 and 45;
- time integration is run on line 48;
- low-pass filtering and downsampling are carried out in section `Low-pass filtering and downsampling` on line 51;
- the edge effects of the low-pass filter are addressed by adding an extra period during time integration (see lines 28 and 29) and removing it afterwards (see lines 63, 64 and 65).

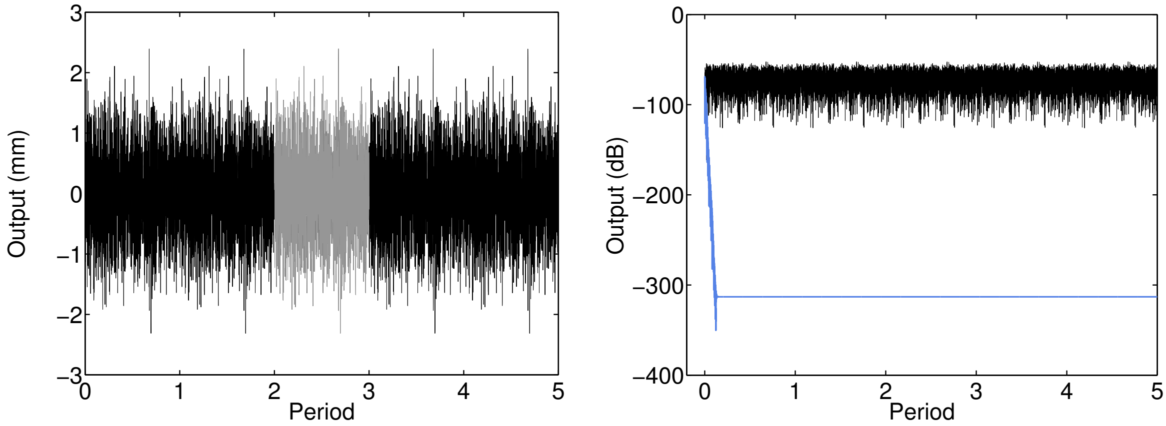
Note that Gaussian noise band-limited in $0 - 375$ Hz is automatically added to the synthesised measurement of $y(t)$ considering a root-mean-squared amplitude of $8\ 10^{-3}$ mm. This provides a realistic signal-to-noise ratio of about 40 dB at 50 N excitation level. The input time series $u(t)$ is assumed to be noiseless.

Fig. 2 (a) displays the calculated system output. The exponential decay of the system transient response is plotted in Fig. 2 (b) by subtracting the last synthesised period from the entire time record. This graph indicates that transients due to initial conditions only affect the first period of measurement, and that the applied periodic input results in a periodic output. It also demonstrates the high accuracy of the Newmark integration, as the transient response reaches the Matlab precision of -313 dB in steady state. Remark that, in this particular case, no noise was added to the output to focus on integration accuracy.

5 Model test and figure of merit

Two fixed test datasets are provided through the benchmark meeting website: a random phase multisine and a sine-sweep signal. The test datasets are noiseless and a sampling frequency of 750 Hz is considered. The random phase multisine dataset contains one steady-state period of 8192 samples. The excited band encompasses all frequencies in 5 – 150 Hz, and the RMS input value is 50 N. The sine-sweep dataset is not in steady state, the simulation started with initial conditions equal to zero. In this case, the amplitude of the input is 40 N, and the frequency band from 20 to 50 Hz is covered at a sweep rate of 10 Hz/min. These test sets function as a target for the obtained model, the model should perform as good as possible on these test datasets. The goal of the benchmark is to estimate a good model on the estimation data. The test data should not be used for any purpose during the estimation.

We expect all participants of the benchmark to report the following figure of merit for all



(a) (b)

Figure 2: System output calculated in response to a multisine input band-limited in 5 – 150 Hz. (a) Output over 5 periods, with one specific period highlighted in grey; (b) output in logarithmic scaling (in black) and decay of the transient response (in blue).

test datasets to allow for a fair comparison between different methods:

$$\boldsymbol{e_{RMSt}} = \sqrt{1/N_t \sum_{t=1}^{N_t} (y_{mod}(t) - y_t(t))^2}, \tag{6}$$

where y_{mod} is the modeled output, y_t is the output provided in the test dataset, N_t is the total number of points in y_t.

Also mention whether the modeled output y_{mod} is obtained using **simulation** (only the test input u_t is used to obtain the modeled output $y_{mod}(t) = F(u_t(1), \ldots, u_t(t))$) or **prediction** (both the test input u_t and the past test output y_t are used to obtain the modeled output $y_{mod}(t) = F(u_t(1), \ldots, u_t(t), y_t(1), \ldots, y_t(t-1))$). Provide both figures of merit (simulation and prediction) if the identified model allows for it.

6 Nonlinear system identification challenges

We anticipate the Bouc-Wen benchmark to be associated with 4 major nonlinear system identification challenges:

- it possesses a nonlinearity featuring memory, *i.e.* a dynamic nonlinearity;
- the nonlinearity is governed by an internal variable $z(t)$, which is not measurable;

- the nonlinear functional form in Eq. (3) is nonlinear in the parameter ν;
- the nonlinear functional form in Eq. (3) does not admit a finite Taylor series expansion because of the presence of absolute values.

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